CHAPTER 1

PROBLEM SOLVING

WITH

ADDITION AND SUBTRACTION

OF

WHOLE NUMBERS
Chapter 1

DEFINITIONS

Definition: **Natural or Counting Numbers**: 1, 2, 3,…

Trick: “It’s natural to count from 1”

Definition: **Whole numbers**: 0, 1, 2,…

The list of whole numbers is almost identical to the list of the natural numbers, except the whole numbers list includes the number ZERO.

Trick: “The whole numbers start with the number that looks like a hole – ZERO.

NOTE: Both the natural numbers and the whole numbers continue on forever (infinitely).

Definition: **Digits**: The 1st 10 whole numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Trick: You dial 7-digit phone numbers on your telephone
You have a 3-digit area code for your phone number
You have a 5-digit zip code

Answer the following questions:

1. What is the largest whole #?
2. What is the smallest whole #?
3. What is the largest natural #?
4. What is the smallest natural #?
5. What is the largest digit?
6. What is the smallest digit?

NUMBER LINES

A number line is the most basic form of a graph.

A number line provides a way of picturing numbers.
INEQUALITIES

<   >

Definition: **< Less Than**: One number is less than a second number if it is located to the left of the second number on a number line.

Definition: **> Greater Than**: One number is greater than a second number if it is located to the right of the second number on a number line.

Write the following in words:

1) \( 1 < 5 \)
2) \( 3 < 12 \)
3) \( 72 > 19 \)

4) Knowing the three previous facts, what else can you conclude about each fact?

5) A, B, and C are 3 arbitrary numbers on the following number line. Please write as many greater than and less than relationships as you can see.

![Number Line]

C   A   B

A look ahead or When will I ever need this?

Knowing how to easily change from < to > sentences can be very helpful in algebra and courses that use these symbols. One will see this in courses like Finite Mathematics, which is often required for Business majors.
Large numbers are organized in three digit groups, separated by commas. Each group has a name and each place in the group has a name as well.

Think: **First** name = the name of the **digit**: (zero) one, two, three, four, five, six, seven, eight, nine

**Middle** name = the name of the **place** in a three digit number: (ones), tens, hundreds

**Last** name = **group** name: (units), thousands, millions, billions, trillions

To help remember the group names you can use the following sentence:

- Usually → **Units**
- Things → **Thousands** (In other words, right to left.)
- Make → Millions
- Boys → Billions
- Trip → Trillions

**Ex. A:** Find the place value name for the digit 2 in the number 70,320,000,498.

Start by putting vertical lines through the commas to separate the groups.

Write the group names under each group using the sentence and starting on the **RIGHT** and going **LEFT** (backwards).

Locate the two and determine its “middle name” (ones, tens, or hundreds) and its last name (units, thousands, millions, billions, trillions)

Answer: The 2 is in the ten millions place.
**Ex. B:** Find the digit in the billions place in the number 70,320,000,498.

**NOTE:** When they say billions place they mean the **one** billions place.

Start by putting vertical lines through the commas to separate the groups.

Write the group names under each group using the sentence and starting on the **RIGHT** and going **LEFT** (backwards).

Locate the billions group and find the digit in the ones place.

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<thead>
<tr>
<th>H</th>
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<tbody>
<tr>
<td>7</td>
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<td>0,</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>4</td>
<td>9</td>
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<tr>
<td>B</td>
<td>M</td>
<td>TH</td>
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```

**Answer:** The 0 is in the one billions place.

**Ex. C:** What is the value of the digit 4 in 70,320,000,498?

Start by putting vertical lines through the commas to separate the groups.

Write the group names under each group using the sentence and starting on the **RIGHT** and going **LEFT** (backwards).

Locate the digit 4 in the number.

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<th>H</th>
<th>T</th>
<th>O</th>
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<tbody>
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<td>0</td>
<td>,</td>
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<tr>
<td>3</td>
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<td>0,</td>
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<tr>
<td>0</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>B</td>
<td>M</td>
<td>TH</td>
</tr>
</tbody>
</table>
```

**First** name: Four  
**Middle** Name: Hundred  
**Last** Name: (Units) The group name is in parentheses because we do not say or write the group name **units**.

**Answer:** The value of the 4 in 70,320,000,498 is **four hundred**.

**NOTE:** We do not say the group name for the **UNITS** group.
1. In the number 9,087,654,321 give the place value name of the following digits
   a) 8
   b) 5
   c) 3
   d) 4
   e) 9
   f) 6

2. In the number 123,789,456 give the digit that represents the following places:
   a) Ten millions
   b) Millions
   c) Hundred thousands
   d) Thousands
   e) Hundreds
   f) Ones
3. Give the value of the digit 6 in each number:
   a) 806,152
   b) 28,261,309,117
   c) 652,000,802,198
   d) 82,922,067

4. Give the value of the digit 9 in each number:
   a) 3,920,458
   b) 75,286,129,003
   c) 89,480,153
   d) 425,903
STANDARD NOTATION TO EXPANDED NOTATION:

We use the place value system for numbers. It states that each digit in any number has a value that depends on its place in that number.

Def: Expanded Notation: The sum of the values of the non-zero digits in a whole number.
Def: Standard Notation: A number written using digits.

To explain the value for each individual digit, separately, in any number we use Expanded Notation as follows:

**Ex. A:** 1706 is a 4-digit number: 1 7 0 6

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>700</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

So… 1706 is 1000 + 700 + 6 in expanded notation.

Standard Notation Expanded Notation

You can write expanded notation for 1706 with a different set up. This works particularly well for large numbers. Another way to “see” expanded notation is as follows:

1706 is 1000 + 700 + 6 is in expanded notation. **Note:** 1706 has three non-zero digits, so there are only three parts to its expanded notation. The tens digit is a zero (a place holder) and zero adds nothing to the overall value, so we skip zeros.

**Ex. B:** 12,000,070,000,604 = 10,000,000,000,000 + 2,000,000,000,000 + 70,000,000 + 600 + 4 in expanded notation

OR 12,000,070,000,604 is 10,000,000,000,000 + 2,000,000,000,000 + 70,000,000 + 600 + 4 in expanded notation

Write in Expanded Notation:

1) 427 2) 3,000,120

3) 90,001,034 4) 18,000,000,502,000
EXPANDED NOTATION TO STANDARD NOTATION

Given a number in expanded notation, we must be able to write the number in standard notation. This is also a good way to CHECK YOUR WORK!!!

Notice, in the previous problems, the first value in the expanded form of a number has the same number of digits as the standard form of the number.

\[
1706 = 1000 + 700 + 6
\]

NOTE: These are both 4 digit numbers.

To change from expanded to standard notation:

1) Count the number of digits for the first value in the expanded form of the number.

2) Set up your answer (standard notation) by writing as many blanks as there are digits in the first value of the expanded notation, including commas.

3) Starting on the right, number all the blanks in the answer.

4) Count the number of digits in each piece of the expanded notation. Write this number under each nonzero digit in the expanded form of the number. This indicates where that digit is placed in the answer.

5) Fill in any empty blanks with zeros.

**Ex. A:**

\[
\begin{align*}
1000 & \quad + \\
700 & \quad + \\
6 & \quad \ \ \\
\hline
1706 & \quad \quad \\
\end{align*}
\]

1) The answer will be a 4-digit number.

2 & 3) Draw 4 dashes and write the numbers 1 through 4 under the blanks from right to left.

4) Place the 1 over dash number 4.

4) Place the 7 over dash number 3.

4) Place the 6 over dash number 1.

5) Fill in any empty blanks with zeros. (Place holder)

Answer: 1000 + 700 + 6 is 1706 in standard notation.
10 Chapter 1

**Ex. B:** \(10,000,000,000,000 + 2,000,000,000,000 + 70,000,000 + 600 + 4 = \)

1) The answer is 14 digits long.
2) Write 14 blanks separated by properly placed commas (to show correct groupings).
3) Number the blanks from right to left.
4) Place the non-zero digits in their places.
5) Fill in any empty blanks with zeros (place holders).

\[
\begin{array}{cccccccccccc}
1 & 2 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 6 & 0 & 4 \\
14 & 13 & 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1
\end{array}
\]

Answer: \(12,000,070,000,604\) is in standard notation

Write the following in Standard Notation:

1) \(200 + 30 + 7\)

2) \(5000 + 700 + 80 + 4\)

3) \(80,000,000 + 60,000 + 3000 + 10\)

4) \(700,000,000,000 + 5,000,000,000 + 20,000,000 + 30,000 + 600\)

**A look ahead** or **When will I ever need this?**

Knowing how to easily change back and forth between expanded and standard notation can be very helpful in courses that use different number systems, like binary or hexadecimal. One will see this in courses like Math Topics (MAT100) and some computer courses.
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
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<tbody>
<tr>
<td>1) Natural numbers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Whole Numbers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Digits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Less than</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Greater than</td>
<td></td>
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</tr>
<tr>
<td>Term:</td>
<td>Official Definition:</td>
<td>In your own words and/or example:</td>
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<tr>
<td>6) Standard notation</td>
<td></td>
<td></td>
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<tr>
<td>7) Expanded notation</td>
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<tr>
<td>8) Place value</td>
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</tbody>
</table>

**Answer the following questions using complete sentences:**

9) Is every whole number a natural number?

10) What is the difference between the natural numbers and the whole numbers?

11) What is the difference between the whole numbers and the digits?

12) What is the smallest 4-digit number?

13) What is the largest 5-digit number?
For questions 14 – 18, use the following number line:

```
<----------0----------b----------2----------3----------a----------5----------d----------7----------8----------c---------->
```

<table>
<thead>
<tr>
<th>For questions 14 – 15, find the following values:</th>
<th>For questions 16 – 18, insert the correct symbol &lt;, &gt;, = between the following pairs of numbers:</th>
</tr>
</thead>
<tbody>
<tr>
<td>14) ( a = )</td>
<td>16) ( b ) ( a ) 17) ( c ) ( d ) 18) ( c ) ( 9 )</td>
</tr>
<tr>
<td>15) ( c = )</td>
<td></td>
</tr>
</tbody>
</table>

For quest., 19 – 21, write in expanded notation: For quest., 22 – 24, write in standard notation:

| 19) \( 52,012,803 \)                           | 22) \( 10,000,000 + 4,000,000 \)              |
|                                               | \( + 5000 + 900 + 20 + 7 \)                   |
| 20) \( 13,000,006,025 \)                       | 23) \( 80,000,000,000 \)                      |
|                                               | \( + 6,000,000,000 + 10,000 \)                |
|                                               | \( + 3000 + 40 \)                             |
| 21) \( 702,030,000,450,000 \)                 | 24) \( 5,000,000,000,000 \)                   |
|                                               | \( + 800,000,000 + 2,000,000 \)              |
|                                               | \( + 4000 + 7 \)                             |
Chapter 1.2

STANDARD NOTATION TO WORD NAMES

Sometimes it is necessary to write numbers out in words. Believe it or not, there are still times when you have to write a check. If you use a credit card to pay tuition, income taxes, and some utility bills, you are charged a fee for the convenience. If the number on the check is improperly written it can void the check, they send it back to you, and now you can be charged a hefty late fee, both by your bank and the recipient’s bank, as well as a late fee.

If you can read a one, two, or three-digit number and you know your group names then you can write the word name for any number no matter how large.

The one – digit numbers are 0 - 9.
The two – digit numbers are 10 - 99.
The three - digit numbers are 100 - 999.

TO WRITE WORD NAMES FOR THREE DIGIT NUMBERS

Word Names for one and two – digit numbers are not a problem. Three – digit numbers are mispronounced by people often. The word “AND” should NOT be inserted to connect the hundreds digit and the rest of the number.

For example, the number 205 is often said as two hundred AND five which is incorrect. The correct way to say this is “two hundred five.” You have to break the bad habit of misreading the word name for three-digit numbers.

To write the CORRECT word name for a three-digit number:

1) Read/Write the digit in the hundreds place followed by the word “hundred”.
2) Read/Write the remaining one or two-digit number.

For example, write the word names of the following:

Ex. A: 412 = four hundred twelve in words.

Ex. B: 907 is nine hundred seven in words.

NOTE: There are NO “ANDS” or digits written in word names for WHOLE NUMBERS.
WRITING WORD NAMES FROM STANDARD NOTATION

1. Start by putting vertical lines through the commas to separate the groups.

2. Write the group names under each group of numbers using the HINT sentence “Usually Things Make Boys Trip.” Start on the RIGHT and work to the LEFT (backwards).

   Remember, you must treat group names like last names, so it helps to write group names near the commas at the end of each group (except “units” which we don’t say or write).

   Usually \( \rightarrow \) Units

   THings \( \rightarrow \) THousands

   Make \( \rightarrow \) Millions

   Boys \( \rightarrow \) Billions

   TRip \( \rightarrow \) Trillions

   Trillions,\( \rightarrow \) Billions,\( \rightarrow \) Millions,\( \rightarrow \) Millions,\( \rightarrow \) THousands,\( \rightarrow \) THousands,\( \rightarrow \) (Units)

   (In other words, right to left – see below.)

3) Start on the left and READ/WRITE the one, two, or three-digit number followed by its group name.

4) Repeat the process for each group until the end of the number. Be sure to skip any group containing all zeros, they add nothing to the word name.

\[
9,122,012,825,716
\]

9,122,012,825,716 is nine trillion, one hundred twenty-two billion, twelve million, eight hundred twenty-five thousand, seven hundred sixteen in words.

**NOTE:** Never write “and” in a whole number name!

“And” is reserved for the decimal point (which is not used in whole numbers).

Never write “Units” (the group name for the last 3-digit group of numbers).

Ex. A: \( 47,308 = \) forty-seven thousand, three hundred eight in words.

\[
\begin{array}{c|c|c|c|c|c|c}
9 & 122 & 012 & 825 & 716 \\
\hline
\text{Trillions} & \text{Billions} & \text{ Millions} & \text{Millions} & \text{ Thousands} & \text{(Units)}
\end{array}
\]

Ex. B: \( 27,032,001,209 = \) twenty-seven billion, thirty-two million, one thousand, two hundred nine in words.
Write a word name for each number: (Write in group names!!)

1) 75,000,013

2) 8,043,000,108

3) 5,000,000,643

4) 800,000,402,025,000

A look ahead or When will I ever need this?

Being able to read word names aloud can be very important in Business and Finance courses, as well as in the Business World, where you may need to say the names for very large numbers.
WRITING STANDARD NOTATION FROM WORD NAMES

If you can write a word name from standard notation you can write standard notation from a word name. In word problems numbers are sometimes written as words, not digits, so you will have to translate the words to standard notation.

Writing Standard Notation from Word Names:

Step 1) Set up 15 dashes to represent five possible groups of three – digit numbers separated by commas. (See below.)

Step 2) Write the group names under each group. (Remember to work backwards.) (See below.)

Step 3) Write the word name for the number in the correct group. The group name follows the number name. (If you need to, circle the number with its group name to make longer word names easier to handle.) (See below.)

Step 4) Fill in any blanks with zeros. *Remember, an empty group requires 3 zeros*(See below.)

Ex.A: Write standard notation for two hundred eighty billion, two hundred million, seven hundred twenty-one thousand, three hundred ninety-six.

Step 1) Two hundred eighty is 280 in digits and goes into the billions group.
Step 2) Two hundred million is 200 in digits and goes into the millions group.
Step 3) Seven hundred twenty-one thousand is 721 in digits and goes into the thousands group.
Step 3) Three hundred ninety-six is 396 in digits and goes into the (units) group. (Remember: the units group name is never written.)

\[
\begin{array}{ccccccccc}
\text{TR} & \text{B} & \text{M} & \text{TH} & \text{U} \\
2 & 8 & 0 & , & , & , & , & , \\
\end{array}
\]

Answer: Two hundred eighty billion, two hundred million, seven hundred twenty-one thousand, three hundred ninety-six is 280,200,721,396 in standard notation.
Ex.B: Write ten trillion, nine hundred seventy thousand, one in standard notation.

Step 1) Ten trillion is 010 as a three digit number, but because it is the first group of numbers (highest group) we leave off the leading zero (first zero).

Step 2) Nine hundred seventy thousand is 970 in digits and goes into the thousands group.

Step 3) One is 001 as a three digit number, and since it is NOT the first number (highest group), write it as 001 and place it in the units group.

Step 4) Fill in the blanks with zeros. (Remember to fill in any blank groups with 3 zeros.)

Answer: ten trillion, nine hundred seventy thousand, one is 10,000,000,970,001 in standard notation.

NOTE: The number 3 written as a three digit number is 003*
The number 27 written as a three digit number is 027*
The number 562 is already a three digit number.

NOTE: Adding zeros in front of a whole number does not change its value.
Write each number in standard notation: (Write in group names!!)

1) Six hundred forty-five billion, eighty million, three hundred ninety-seven thousand, two hundred fifty-six

2) Five trillion, forty-eight million, six hundred five

3) One hundred eighty-one million, three thousand, nine hundred forty-four

4) Twenty-one trillion, seven hundred seventy-seven thousand, eleven

A look ahead or When will I ever need this?

Often in Business and Finance courses, as well as in the Business world, you will hear large numbers spoken that are necessary for related computations. You will need to be able to write standard notation for the numbers you are hearing before you can compute with them.
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<tbody>
<tr>
<td>1) Group names</td>
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<td>2) Place value</td>
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<td>3) Standard notation</td>
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<tr>
<td>4) Expanded notation</td>
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For questions 5 – 7, put in the commas and corresponding group names for the following numbers:

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<td>7</td>
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<td>9</td>
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For questions 8 – 10, write word names for each of the numbers:

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<tr>
<td>8)</td>
<td>201,012,185</td>
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<tr>
<td>9)</td>
<td>70,083,000,009</td>
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<tr>
<td>10)</td>
<td>46,000,832,012,000</td>
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</table>
For questions 11 – 13, write the following word names in standard notation:

11) Three hundred twelve billion, one hundred eighty-six million, five hundred twenty-nine thousand, seven hundred sixteen.

12) Eight billion, twelve million, seven thousand.

13) Seven trillion, seventeen thousand, ninety-one.

For questions 14 – 16, give the place value name of the digit 0 in the following numbers:

14) 36,052

15) 413,206,759

16) 6,320,438,526,924

For questions 17 – 20, indicate the digit that represents the following place values in the following number: 1,234,567,890,946

17) Thousands

18) Ten billions

19) Ones

20) Hundred millions
Chapter 1.3

ADDITION CONCEPTS AND PROPERTIES

Definition: **Sum** (noun): the result of an addition problem

**Sum** (verb): Add two or more numbers.

We can add across from left to right or we can add vertically from top to bottom:

\[ 3 + 4 = 7 \]

or we can use the number line to add:

[Diagram: Number line showing 3 + 4 = 7]

**NOTE**: Number line addition is a move to the right.

**NOTE**: Know your basic addition facts, adding any two one-digit numbers. (See page 465.) Speed and accuracy are important.

Properties of Addition (Laws)

**Commutative Law**: \[ a + b = b + a \]

\[ 3 + 4 = 4 + 3 \]

**Associative Law**: \[ (a + b) + c = a + (b + c) \]

\[ (3 + 4) + 5 = 3 + (4 + 5) \]

**Identity Law or Property of Zero**: \[ a + 0 = a \]

\[ 3 + 0 = 3 \]

Answer the following questions:

1) What law is illustrated? \[ 3 + 0 = 0 + 3 \]

2) Complete the statement using the Associative Law: \[ 2 + (8 + 9) = \] ________________

3) Find the sum of the digits.
MIXED UNIT CONVERSIONS AND ADDITION

I am 63 inches tall, but I never say that. I say I am 5’ 3” tall. I would never say I was 4’ 15” tall, which is ridiculous.

Keep in mind that 1 foot = 12 inches. Every time I reach 12 inches I change it to 1 foot.

**For example:** Convert 4’ 15” to the proper way of expressing feet and inches.

Step 1) Rewrite the 15” as 12” + 3”, since 12” + 3” = 15”

Step 2) Rewrite the 12” as 1’, since 1’ = 12”

Step 3) Now use the Associative Property (move parentheses):

Now we can add the 4’ + 1’ = 5’

Answer: 4’ 15” = 5’ 3”

**Some conversions you need to know:**

| Distance: NOTE: feet = ’ inches = ” |
|---|---|
| 1 foot = 12 inches (1’ = 12”) | 1 yard = 3 feet (1 yd = 3’) |

<table>
<thead>
<tr>
<th>Time:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 minute (min) = 60 seconds (sec)</td>
</tr>
<tr>
<td>1 hour (hr) = 60 min</td>
</tr>
<tr>
<td>1 day = 24 hrs</td>
</tr>
<tr>
<td>1 week (wk) = 7 days</td>
</tr>
<tr>
<td>1 year (yr) = 52 wks</td>
</tr>
<tr>
<td>1 yr = 365 days (except for Leap Year!)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weight:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pound (lb) = 16 ounces (oz)</td>
</tr>
</tbody>
</table>

**Ex.A:** Convert 3 years 18 months to the proper way of expressing years and months.

Step 1) Write the conversion rule above the problem.  

Step 2) Break the smaller units into the maximum number of smaller units plus any leftover smaller units.

Step 3) Convert the maximum number of smaller units to 1 of the larger units. (Use Associative Property.)

Step 4) Add the larger units together.

Answer: 3 years 18 months = 4 years 6 months

**NOTE:** You only need to apply the conversion rule when the number for the smaller unit of measurement (months) is large enough (12 months or more) to apply the conversion rule!
Two Digit addition:

\[
\begin{align*}
47 & \quad 4 \text{ tens} + 7 \text{ ones} \\
+39 & \quad +3 \text{ tens} + 9 \text{ ones} \\
86 & \quad 7 \text{ tens} + 16 \text{ ones} \quad \text{(Now convert)}
\end{align*}
\]

Step 1) Write 16 ones = 10 ones + 6 ones \quad \rightarrow \quad 7 \text{ tens} \quad + \quad (10 \text{ ones} + 6 \text{ ones})

Step 2) Convert 10 ones to 1 ten: \quad \rightarrow \quad 7 \text{ tens} \quad + \quad (1 \text{ ten} \quad + \quad 6 \text{ ones})

Step 3) Use Associative Property: \quad (7 \text{ tens} + 1 \text{ ten}) + 6 \text{ ones} = 8 \text{ tens} + 6 \text{ ones}

\[
\begin{align*}
86 & = \quad 8 \text{ tens} + 6 \text{ ones}
\end{align*}
\]

NOW, what if the original problem was 4 feet 7 inches + 3 feet 9 inches?

Mixed Unit addition:

- 4 feet 7 inches
- + 3 feet 9 inches
- \rightarrow \quad 7 \text{ feet} + 16 \text{ inches}

No one would ever say 7 feet 16 inches, that’s ridiculous. So use the technique you just learned to convert the 7 feet 16 inches to 8 feet 4 inches as follows:

Step 1) Write the conversion rule above the problem.

\[
\begin{align*}
1 \text{ foot} = 12 \text{ inches}
\end{align*}
\]

Step 2) Break the smaller units into the maximum number of smaller units plus any leftover smaller units.

\[
\begin{align*}
7 \text{ feet} & + (1 \text{ foot} + 12 \text{ inches} + 4 \text{ inches})
\end{align*}
\]

Step 3) Convert the maximum number of smaller units to 1 of the larger units.

(Use Associative Property.)

\[
\begin{align*}
(7 \text{ feet} + 1 \text{ foot}) + 4 \text{ inches of}
\end{align*}
\]

Step 4) Add the larger units together.

\[
\begin{align*}
8 \text{ feet} & + 4 \text{ inches}
\end{align*}
\]

Answer: 7 feet 16 inches = 8 feet 4 inches

A “simpler” way to write this is as follows:

1 foot = 12 inches

Step 1) Add straight down the columns but do not carry!

Notice: 16 inches is too big!

Step 2) Take out the 12 inches and apply the conversion rule. The 12 inches becomes 1 foot.

Step 3) 12 in. comes out of the inches column and goes back in to the foot column as 1 foot.

Answer: 8 feet 4 inches is the sum.
Mixed Unit Problems

a) \(4 \text{ lbs } 7 \text{ oz} + 3 \text{ lbs } 9 \text{ oz}\) THINK: What is the connection between pounds and ounces?

b) \(4 \text{ years } 7 \text{ months} + 3 \text{ years } 9 \text{ months}\) THINK: What is the connection between years and months?

c) \(4 \text{ weeks } 5 \text{ days} + 3 \text{ weeks } 6 \text{ days}\) THINK: What is the connection between weeks and days?

d) \(4 \text{ hours } 35 \text{ minutes} + 3 \text{ hours } 46 \text{ minutes}\) THINK: What is the connection between hours and minutes?

e) \(4 \text{ days } 13 \text{ hours} + 3 \text{ days } 17 \text{ hours}\) THINK: What is the connection between days and hours?

A look ahead or When will I ever need this?

Planning on doing any scheduling at work…for the day, the week, the month? Planning on riding on the LIRR or the bus? These, as well as many kinds of measurements in carpentry or cooking, etc., require the ability to manipulate numbers that often have different units of measurement.
COUNTING TRIANGLES

Definition - A triangle is a 3-sided closed figure with 3 angles (thus a tri-angle).

In this course you will be asked to count the number of triangles in a given design.

For example: This design is made up of several puzzle pieces which we call bits. There are many different triangles in this design. In this reading you will find an explanation of how to count triangles, several examples and then a design for you to work with.

Throughout these pages a column is provided at the right. In this column please write at least 5 questions, comments and/or questions.

Example 1

How many triangles do you see? (No, this is not a trick!)
Yes, there is one triangle here.

Example 2

Now, how many triangles do you see? If you said two triangles, then you are correct.

Notice the small triangle on top and the larger triangle made from the two pieces in the figure.

This figure is made up of 2-bits or 2-pieces. Since the small triangle is only one of the two pieces, it is called a 1-bit triangle. The larger triangle is made from both pieces so it is called a 2-bit triangle.

Example 3

The following figure is also made up of 2 pieces.

Notice the 1-bit triangle on the left side and the 1-bit triangle on the right side.

So in this figure there are two 1-bit triangles. There is also the large triangle made from both pieces. This is a 2-bit triangle.

So, there are three triangles in the given figure.

Since these figures are simple (they are made up of few "puzzle" pieces) it is easy to just look at them and count triangles. As the figures become more complex, the following method can be helpful in organizing your information. This method will help ensure that you do not miss any triangles or count any triangle more than once.
Consider the figure \( \triangle \). Since it is made from 4 "puzzle" pieces you could have triangles made from 1-piece, 2-pieces, 3-pieces or 4-pieces. These are the types of triangles you will look for.

1. To make things easier to keep track of, label the pieces with letters. \( \triangle ab \)

2. Now list the types of triangles to look for. [Notice that "piece" and "bit" are used in the same way. "Bit" is just easier to write.]

\[
\begin{array}{c|c}
 \text{Type of triangle} & 1\text{-bit} \\
 & 2\text{-bit} \\
 & 3\text{-bit} \\
 & 4\text{-bit} \\
\end{array}
\]

3. Now start looking for triangles in the figure. First look for 1-bit triangles then 2-bit triangles, etc. until you have tried each type.

Remember: a 1-bit triangle is a single piece of the puzzle that is also a triangle.

A 2-bit triangle is a single triangle that is made up of two puzzle pieces, a 3-bit triangle is a single triangle that is made up of three puzzle pieces, etc.

\[
\begin{array}{c}
 a \\
 b \\
 c \\
 d \\
\end{array}
\]

As we look at each, notice that only \( \triangle @ \) and \( \triangle \) are triangles. So \( @ \) and \( \) are 1-bit triangles. You would record these in the following manner:

\[
\begin{array}{c|c|c}
 \text{type of triangle} & \text{names of triangles} & \text{number of triangles} \\
 1\text{-bit} & @ & 2 \\
 2\text{-bit} \\
 3\text{-bit} \\
 4\text{-bit} \\
\end{array}
\]

\[
\text{total = } \_\text{triangles}
\]
When counting triangles made from more than one-piece, you should look for the following structures:

a. **Top-down** - triangles formed by pieces that are stacked one on top of the other as in Example 2 on page 1.

b. **Side-by-Side** - triangles formed by pieces that are next to each other as in Example 3 on page 1.

c. **Cluster structure** - triangles formed by bunches of pieces that are not top-down or side-by-side.

   *ex* - this is a 3-bit triangle (cluster structure)

   *ex* - this is a 2-bit triangle (cluster structure)

And now, back to our triangle. We are now looking for 2-bit triangles.

**Top-down structure:**

\[\triangle a \quad \text{and} \quad \triangle b\]

\(\triangle a\) and \(\triangle b\) are two 2-bit triangles.

Notice here that I use a "pendulum swing." Once I see the \(\triangle a\) triangle on the left I swing the pendulum one place to the right and look for another triangle. Then see the \(\triangle b\) triangle. If I swing the pendulum again I fall off the edge of the triangle. (So it is time to move on.)

**Side-by-side structure:**

\[\triangle ab\]

\(\triangle ab\) is also a 2-bit triangle.

[Notice that \(\triangle cd\) is a 2-bit figure (side-by-side structure) but it is not a triangle.]

Now our chart looks like this:

<table>
<thead>
<tr>
<th>type of triangle</th>
<th>names of triangles</th>
<th>number of triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-bit</td>
<td>(a)</td>
<td>2</td>
</tr>
<tr>
<td>2-bit</td>
<td>(a\ b)</td>
<td>3</td>
</tr>
<tr>
<td>3-bit</td>
<td>(a\ b\ c)</td>
<td></td>
</tr>
<tr>
<td>4-bit</td>
<td>(ab\ c)</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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If we look for 3-bit triangles, we cannot find any:

(Notice that these shapes are cluster structure.)

Finally, we look for 4-bit triangles.

abcd is a 4-bit triangle - cluster structure.

So now our chart looks like this:

<table>
<thead>
<tr>
<th>type of triangle</th>
<th>names of triangles</th>
<th>number of triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-bit</td>
<td>(a) (b)</td>
<td>2</td>
</tr>
<tr>
<td>2-bit</td>
<td>ac bd ab</td>
<td>3</td>
</tr>
<tr>
<td>3-bit</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>4-bit</td>
<td>abcd</td>
<td>1</td>
</tr>
</tbody>
</table>

The last step is to find the sum of all the triangles we have counted far.

2 + 3 + 0 + 1 = 6 triangles

Problems

In the following figures you are to fill in the empty column in the chart.

**Problem I.** How many triangles are there?

<table>
<thead>
<tr>
<th>type of triangle</th>
<th>names of triangles</th>
<th>number of triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-bit</td>
<td>a b c</td>
<td></td>
</tr>
<tr>
<td>2-bit</td>
<td>ab bc</td>
<td></td>
</tr>
<tr>
<td>3-bit</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Notice the pendulum swing.
Problem II. How many triangles are there?

<table>
<thead>
<tr>
<th>type of triangle</th>
<th>names of triangles</th>
<th>number of triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-bit</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>2-bit</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>3-bit</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>4-bit</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>5-bit</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>6-bit</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

total = 9 triangles

Now that you have had some practice, solve the following.....

Problem III. How many triangles are there?

<table>
<thead>
<tr>
<th>type of triangle</th>
<th>names of triangles</th>
<th>number of triangles</th>
</tr>
</thead>
</table>

|                |                    |                    |

total = ___ triangles

A look ahead or When will I ever need this?

Counting triangles helps you organize your thinking. This particular skill is helpful in life, as well as when writing sample spaces for probability problems in a Statistics course (MAT102). Statistics are used in all of the social sciences and medical fields. Many students, including those majoring in Business and Nursing, take statistics.
Homework 1.3  Name: 

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Sum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Commutative Law of Addition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Associative Law of Addition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Identity Property of Zero</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Mixed Unit Problems</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6) Sum 8 and 9

8) List all the pairs of numbers that give the sum of 10:

<table>
<thead>
<tr>
<th>7) What should be added to the digit 3 to get a sum of 3?</th>
</tr>
</thead>
<tbody>
<tr>
<td>6) Sum 8 and 9</td>
</tr>
</tbody>
</table>

For questions 9 – 11, find the following sums:

<table>
<thead>
<tr>
<th>9) 81 + 42 + 67</th>
</tr>
</thead>
<tbody>
<tr>
<td>10) 2 + 3 + 8 + 6 + 0 + 4</td>
</tr>
<tr>
<td>11) 403 + 758 + 281 + 46</td>
</tr>
</tbody>
</table>

For questions 12 – 15, find sums for the following mixed unit problems:

<table>
<thead>
<tr>
<th>12) 4 years 8 months + 2 years 7 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>13) 4 feet 3 inches + 2 feet 5 inches</td>
</tr>
<tr>
<td>14) 2 hours 22 minutes + 6 hours 48 minutes</td>
</tr>
<tr>
<td>15) 7 pounds (lbs) 11 ounces (oz) + 6 pounds (lbs) 9 ounces (oz)</td>
</tr>
</tbody>
</table>
For questions 16 – 17, figure out how many triangles are in each picture:

16) Type | Name | Number

17) Type | Name | Number

18) There are 5 tennis players in a tournament. If each player must play all the other player exactly once, how many matches will be played?
Challenge Problem:

The Judge Problem

There are ten judges in a room. Each judge shakes all of the other judges’ hands exactly once. How many handshakes are there?
Rounding and estimating

You round individual numbers.

Rounding is a tool, generally used to estimate answers.

In an estimate you use rounded numbers to get an idea of what an exact answer may be near.

People often use an estimate when standing in line in a store trying to decide if they have enough money to buy what they want when they only have cash in their pocket.

That is...

To estimate a sum:

First round all numbers to the same (highest) place value, and then add to get the estimated sum.

NOTE: If you add the exact numbers first, you get an exact answer (an exact sum)...when you round that answer, you get a rounded sum, NOT an estimated sum.

NOTE: When doing estimates, ROUND FIRST!!

(You are NOT interested in exact details.)

REMEMBER - estimates are supposed to be quick and easy!!

Rounding Rules

To round any whole number:

1. **Locate** the digit in the place value you are rounding to

2. **Mark it** by dropping a vertical line/wall to the right of the digit.

3. **Look** for the digit just to the right of the wall. This digit will “tell” you what to do.

4. **Decide**: a) If the digit just to the right of the wall is: 0, 1, 2, 3, or 4 the entire number on the left of the vertical line/wall stays the same.

   b) If the digit just to the right of the wall is: 5, 6, 7, 8, or 9 **add 1** to the entire number on the left of the vertical line/wall.

5. **Finish**: In either case, all the digits to the right of the vertical line/wall become zeros.
ROUNDING WHOLE NUMBERS

Ex.A: Round 1547 to the hundreds place (to the nearest hundred).

Step 1) LOCATE:
(Write the number 100 above the number you are given.)

Step 2) MARK IT:
(Drop a vertical line between the “1” and the zeros.)

Step 3) LOOK:
Find the digit to the right of the line. (Here it’s the 4.)

Step 4) DECIDE:
Leave the 15 alone or add a 1 to the 15?
The digit on the right is a 4, which says leave the 15 alone.

Step 5) FINISH:
Here change the 4 and 7 to zeros and you are done.

Answer: 1547 is about 1500 when rounding to the 100’s place (to the nearest 100).

NOTE: Notice we rounded to the hundreds place and the answer has two zeros to the right of the wall. When skip counting by hundreds, 0, 100, 200, etc. this answer makes sense.
Solve the following problems:

1) Round 219, 875 to the nearest 10
2) Round 219, 875 to the nearest 100

3) Round 219, 875 to the nearest 1000
4) Round 219, 875 to the nearest 10,000

5) Estimate the sum by rounding to the \textbf{nr. 10}
6) Estimate the sum by rounding to the \textbf{nr. 100}

\begin{align*}
3,304 & \quad 3,304 \\
7,419 & \quad 7,419 \\
2,122 & \quad 2,122 \\
+ 1,375 & \quad + 1,375
\end{align*}

7) Estimate the sum by rounding to the \textbf{nr. 1000}
8) Estimate the sum by rounding to the \textbf{nr. 10,000}

\begin{align*}
3,304 & \quad 23,304 \\
7,419 & \quad 7,419 \\
2,122 & \quad 12,122 \\
+ 1,375 & \quad + 1,375
\end{align*}
NOTE: When estimating in real life, you are not told a place to use for the rounding, so pick the **highest** place that “works” for most of the numbers: Try to round so that at least ½ of the numbers remain in the problem (that is, they don’t round to zero).

What place would you choose to round the numbers when estimating the following? Why?

1) \[3905 \quad 456 \quad 682 \quad 89 \quad \underline{+702}\]
2) \[21,956 \quad 9,071 \quad 1,312 \quad 843 \quad \underline{+2,435}\]

3) Estimate the sum: \[3,304 \quad 19 \quad 822 \quad \underline{+1,675}\]

4) Let’s go car shopping. Suppose your mom wants a brand new Subaru Outback Premium. It’s her first new car in 31 years, so she decides to get all the extras for the car. The base price is $32,628, the extended warranty for 5 years/100,000 miles is an additional $2,450, the blind spot detection system is an extra $3,789, dealer prep fees are $1578, and taxes are $3463.

A) Approximately how much will she spend on extras?

B) Approximately how much will the car cost?

**A look ahead** or **When will I ever need this?**

Often in the Business world, as well as in your daily life, you will want to know approximately how much of something you will need to purchase or approximately how much a large purchase will cost. Knowing how to estimate will help you quickly & easily figure this out without getting bogged down in the details.
Homework 1.4

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Rounding a number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Place value names</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Sum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Estimating a sum</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 5 – 7 round the given numbers to the following places (or round to the nearest):

<table>
<thead>
<tr>
<th></th>
<th>a) tens place</th>
<th>b) hundreds place</th>
<th>c) thousands place</th>
</tr>
</thead>
<tbody>
<tr>
<td>5) 4388</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) 895</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7) 72,875</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For questions 8 – 10, Estimate the sums of the following numbers by rounding each number to the indicated place value: (Please show all the rounded numbers you are using in the space provided.)

<table>
<thead>
<tr>
<th>tens place</th>
<th>hundreds place</th>
<th>thousands place</th>
</tr>
</thead>
<tbody>
<tr>
<td>8)</td>
<td>9)</td>
<td>10)</td>
</tr>
<tr>
<td>405</td>
<td>356</td>
<td>1352</td>
</tr>
<tr>
<td>87</td>
<td>42</td>
<td>897</td>
</tr>
<tr>
<td>9</td>
<td>859</td>
<td>42</td>
</tr>
<tr>
<td>356</td>
<td>1342</td>
<td>41,388</td>
</tr>
<tr>
<td>+ 95</td>
<td>+ 941</td>
<td>+ 8,722</td>
</tr>
</tbody>
</table>

11) Given the following problem, where you are simply asked to “estimate the sum,”

Estimate: $2,345 + 87 + 943 + 8,795 + 3,562 + 17 + 516$

a) What would you do first?

b) Please explain why you would choose to do the problem this way.

12a) Median salaries in the US

<table>
<thead>
<tr>
<th></th>
<th>In 1965</th>
<th>In 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual median salary in each year:</td>
<td>$6900</td>
<td>$51,017</td>
</tr>
<tr>
<td>Median salary adjusted for inflation to the year 2014:</td>
<td>$52,111</td>
<td>$52,862</td>
</tr>
<tr>
<td>Round the adjusted median salaries for each year to the nearest 10,000:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12b) Now that you have rounded each of the median salaries, adjusted for inflation, to the nearest ten-thousands place, compare and state your conclusion in a complete sentence.
Chapter 1.5

SUBTRACTION CONCEPTS

Definition: **Difference**: The result of a subtraction problem.

We can subtract across from left to right or we can subtract vertically (from top to bottom)

\[ 7 - 4 = 3 \]

or we can use the number line to subtract:

\[ \text{NOTE: Number line subtraction is a move to the left.} \]

7

Check subtraction by writing related addition:

\[ 7 - 4 = 3 \, ? \]

Check: \[ 3 + 4 = 7 \]

**WORDS** to indicate subtraction: “take away” “how many are left?” “find the difference” “how much more?”

Use related sentences to **solve** simple algebraic equations (add 4 to both sides of the equation):

\[ x - 4 = 3 \]

\[ + 4 = +4 \]

\[ x = 7 \]

Answer: \[ x = 7 \]

Check “missing number” problems by the following steps:

1) Rewrite the original problem with ( ) in for the missing number, here it is x
2) Now substitute the value for x (determined by your answer) into the ( )
3) DO THE MATH to see if this value for x makes a true mathematical statement.

Do the Properties of Addition (Laws) work for subtraction?

**Commutative Law**:

\[ 3 + 4 = 4 + 3 \]

\[ 7 = 7 \, ✓ \]

Because: \[ 4 \neq -4 \]

**Associative Law**:

\[ (3 + 4) + 5 = 3 + (4 + 5) \]

\[ 7 + 5 = 3 + 9 \]

\[ 12 = 12 \, ✓ \]

\[ (8 - 4) - 2 = 8 - (4 - 2) \, ? \]

\[ 2 
eq 6 \]

\[ 4 - 2 
eq 8 - 2 \]
More Equation Solving

Solve each equation and check:

1. $13 + X = 20$  
   CHECK:

2. $X - 10 = 49$  
   CHECK:

3. $X + 20 = 57$  
   CHECK:

4. $X + 16 = 61$  
   CHECK:

5. $W - 17 = 53$  
   CHECK:

6. $47 + N = 84$  
   CHECK:
**Missing digit Addition:** These problems ask you to think algebraically to find the correct digits to place in the boxes that make the problem mathematically accurate.

\[
\begin{array}{ccc}
 & & \square \\
+ & 4 & 7 \\
\hline
 & 9 & 3 & \square
\end{array}
\]

Think of basic addition and begin in the ones column:

You have \(3 + \square = 2\) but this is impossible in a whole number world.

Think: the ones digit is 2, but there must be a tens digit, so it must be \(3 + \square = 12\)

Consider basic facts and \(\square = 9\) or you can solve the basic equation with some simple algebra:

\[
\begin{align*}
3 + \square &= 12 \\
- 3 &= -3 \\
\square &= 9
\end{align*}
\]

Now put the 9 in the box in the ones column and REMEMBER to carry the 1 into the tens column!

Now you have:

\[
\begin{array}{ccc}
 & & \square \\
+ & 4 & 7 \\
\hline
 & 9 & 3 & \square
\end{array}
\]

Now work on the tens column:

You have \(1* + 2 + \square = 1\)

(*This 1 is the carry-over from adding in the ones column.) This gives you \(3 + \square = 1\).

Again, this is impossible in a whole number world.

Think: the ones digit is 1, but there must be a tens digit, so it must be \(3 + \square = 11\).

Consider basic facts and \(\square = 8\) or you can solve the basic equation with some simple algebra:

\[
\begin{align*}
3 + \square &= 11 \\
- 3 &= -3 \\
\square &= 8
\end{align*}
\]

Now put the 8 in the box in the tens column and REMEMBER to carry the 1 into the hundreds column!!

Now you have:

\[
\begin{array}{ccc}
 & & \square \\
+ & 4 & 7 \\
\hline
 & 9 & 3 & \square
\end{array}
\]
In the **hundreds column**:  

You have $1* + 7 + ? = 3$  

(*This 1 is the carry-over from adding in the tens column.) This gives you $8 + ? = 3$.  

Again, there must be a tens digit in this sum and this must be $8 + ? = 13$ if we are in a whole number world.

So from basic addition facts $?= 5$. You can also solve the basic equation with some simple algebra:

$$
8 + ? = 13 \\
\text{-} 8 = \text{-} 8 \\
? = 5
$$

Now put the 5 in the box in the hundred’s column and **REMEMBER** to carry the 1 into the thousands column!

Now you have:

```
  1 1 1
+ 4 7 8 9
  9 3 1 2
```  

Finally, in the thousands column we have $1 + 4 + ? = 9$. This gives us $5 + ? = 9$, so $?= 4$.

When you put the 4 in the box in the thousand’s column, the final answer is:

```
  1 1 1
+ 4 7 8 9
  9 3 1 2
```  

**CHECK** Missing Digit problems by doing the problem with the answers **substituted** into the boxes in the **given problem**. See if the addition works! Do you get the same sum?  

Check:

```
  1 1 1
+ 4 7 8 9
  9 3 1 2
```  

A look ahead or When will I ever need this?  

Missing digit problems will help you exercise your brain & help you to think more algebraically. This is an important skill, even if you are not doing this exact type of problem. This is true as well for algebra. Practicing algebraic thinking is like mental calisthenics, exercise for the mind.
Find the **Missing Digits**:

Don’t forget to check your work by substituting answers into the given problem and see if the problem is mathematically accurate. Here, see if the addition works!

1)  

\[
\begin{array}{ccc}
8 & 7 & 9 \\
4 & 0 & \boxed{\phantom{1}} \\
\boxed{\phantom{1}} & 3 & 4 \\
+ & 7 & \boxed{\phantom{1}} & 6 \\
\hline
& & 4 \ 2 \ 0 \\
\end{array}
\]

Check:

2)  

\[
\begin{array}{ccc}
\boxed{\phantom{1}} & 8 & 4 \\
+ & 4 & \boxed{\phantom{1}} & \boxed{\phantom{1}} \\
\hline
& 8 & 7 & 2 \\
\end{array}
\]

Check:

3)  

\[
\begin{array}{ccc}
1 & 2 & \boxed{\phantom{1}} & 5 \\
3 & 6 & 9 \\
7 & 3 & \boxed{\phantom{1}} \\
+ & \boxed{\phantom{1}} & 7 & 3 \\
\hline
& & 2 & 2 & 2 \\
\end{array}
\]

Check:

4)  

\[
\begin{array}{ccc}
3 & 9 & \boxed{\phantom{1}} \\
1 & 5 & 3 \\
\boxed{\phantom{1}} & \boxed{\phantom{1}} & 2 \\
+ & 1 & 9 \\
\hline
& & 5 & 0 \\
\end{array}
\]

Check:
Solve the following Mixed Unit problems:

1) \[ 9 \text{ ft } 3 \text{ in} \]
   \[ +\ 3 \text{ ft } 9 \text{ in} \]

2) \[ 9 \text{ lbs } 3 \text{ oz} \]
   \[ +\ 3 \text{ lbs } 9 \text{ oz} \]

3) \[ 9 \text{ hrs } 36 \text{ min} \]
   \[ +\ 3 \text{ hrs } 39 \text{ min} \]

4) \[ 7 \text{ ft } 10 \text{ in} \]
   \[ +\ 10 \text{ ft } 7 \text{ in} \]

5) \[ 8 \text{ lbs } 11 \text{ oz} \]
   \[ +\ 11 \text{ lbs } 8 \text{ oz} \]

6) \[ 3 \text{ hrs } 47 \text{ min} \]
   \[ +\ 4 \text{ hrs } 54 \text{ min} \]
Homework 1.5

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Sum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Commutative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Law of Addition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Associative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Law of Addition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Digits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term:</td>
<td>Official Definition:</td>
<td>In your own words and/or example:</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>7) Mixed Units</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8) How do you check a subtraction problem? Please show an example:

9a) If you have $7, can you lend a friend $3? Please explain:

9b) If you have $3, can you lend a friend $7? Please explain:

10) We know the commutative law works for addition. Does the commutative law work for subtraction? Please explain:

For questions 11 – 13, solve for x and check your work:

11) \( x + 21 = 47 \)

check:

12) \( x - 17 = 35 \)

check:

13) \( 124 + x = 300 \)

check:
For questions 14 – 16, find sums for the following mixed unit problems:

14)  
8 ft. 1 in.  
4 ft. 2 in.  
3 ft. 5 in.  
+ 1 ft. 3 in.

15)  
3 weeks 4 days  
+ 5 weeks 6 days

16)  
2 hrs. 49 min.  
+ 3 hrs. 35 min.

For questions 17 – 19, solve for the missing digits and check the sums:

17)  
4 \square 
+ \square 8
\square 2 \square

18)  
\square 0 5 
3 \square 8 
+ 8 9 \square 
1 6 5 0

19)  
3 6 2 \square 
4 \square 9 1 
+ 7 8 \square 6 
1 \square, 5 7 5
Chapter 1.6

MULTI-DIGIT SUBTRACTION

Regular Subtraction:

\[
\begin{array}{c}
\phantom{-}74 \\
- \phantom{9}39 \\
\hline
\phantom{-}35
\end{array}
\]

In regular subtraction you start at the ones column and since you can’t take 9 from 4 you “borrow” a ten from the tens column and break it into ones and add the ones to the ones column.

Rewrite:

\[
\begin{array}{c}
74 \\
\rightarrow \\
7 \text{ tens} + 4 \text{ ones}
\end{array}
\]

Rewrite:

\[
\begin{array}{c}
7 \text{ tens} + 4 \text{ ones} \\
\rightarrow \\
(6 \text{ tens} + 1 \text{ ten}) + 4 \text{ ones}
\end{array}
\]

(Associative Property!)

Rewrite:

\[
\begin{array}{c}
6 \text{ tens} + (1 \text{ ten} + 4 \text{ ones}) \\
\rightarrow \\
6 \text{ tens} + (10 \text{ ones} + 4 \text{ ones})
\end{array}
\]

Rewrite:

\[
\begin{array}{c}
6 \text{ tens} + (10 \text{ ones} + 4 \text{ ones}) \\
\rightarrow \\
6 \text{ tens} + 14 \text{ ones}
\end{array}
\]

Original Problem: 74 - 39

This Example with Regrouping:

\[
\begin{array}{c|c|c|c}
\text{Original} & \text{This Example} & \text{Simpler} & \text{Check for} \\
\text{Problem} & \text{with Regrouping} & \text{Regrouping} & \text{Subtraction:} \\
& & (Borrowing) & (Use Addition) \\
74 & 6 \text{ tens} + 14 \text{ ones} & 6 \text{ ones} & 35 \\
-39 & -3 \text{ tens} - 9 \text{ ones} & -39 & +39 \\
35 & 3 \text{ tens} + 5 \text{ ones} & 35 & 74
\end{array}
\]

Answer: The difference between 74 and 39 is 35.

Find the following differences using the Traditional Method:

(Remember to check your work using addition.)

1a) 742 - 521  
2a) 5345 - 2316  
3a) 873 - 185  
4a) 5456 - 2564

check:  
check:  
check:  
check:
SUBTRACTION WITH ADDING UP

Some of you have learned to subtract with the traditional method taught here in the U.S., which is subtraction with borrowing.

Did you know there are many ways to do subtraction? Some involve borrowing but it’s a little different from the traditional method taught here in the U.S.

There is also a method of subtracting where you never need to borrow.

This alternate method is referred to as “adding up.” It is done with algebraic thinking and the check for subtraction, which says if you add the bottom two numbers in the subtraction, you should get the number at the top of the subtraction.

Think: “What number at the bottom (??) + the middle number (39) = the top number (74 = sum)”

Fortunately this does not need to be done all at once. You can do it like regular addition, just upside down as follows:

Starting in the ones column you have $\? + 9 = 4$. No digit works, so here the sum must be 14. We know that $5 + 9 = 14$, so the missing digit in the ones column at the bottom = 5.

Now, when adding up in the tens column you have $1 + ? + 3 = 7$. Simplifying, we get $4 + ? = 7$, so the missing digit in the tens column = 3.

Answer: The difference between 74 and 39 is 35. (The check is: $35 + 39 = 74$; correct!)

Find the following differences by Adding Up (remember to check your work):

1b) 7 4 2 2b) 5 3 4 5 3b) 8 7 3 4b) 5 4 5 6
- 5 2 1 - 2 3 1 6 - 1 8 5 - 2 5 6 4
**MIXED UNIT SUBTRACTION**

What if this regular subtraction problem, \(74 - 39\), actually had labels attached to the numbers?

What if it was: \[\begin{array}{c}
7 \text{ feet} & 4 \text{ inches} \\
-3 \text{ feet} & 9 \text{ inches}
\end{array}\]

The same borrowing that works for regular subtraction works for Mixed Unit Subtraction:

1. **Rewrite:** \[7 \text{ feet } + 4 \text{ inches} \longrightarrow (6 \text{ feet } + 1 \text{ foot}) + 4 \text{ inches} \leftrightarrow (\text{Associative Property!})\]
2. **Rewrite:** \[(6 \text{ feet } + 1 \text{ foot}) + 4 \text{ inches} \longrightarrow 6 \text{ feet } + (12 \text{ inches } + 4 \text{ inches}) \leftrightarrow\]
3. **Rewrite:** \[6 \text{ feet } + (12 \text{ inches } + 4 \text{ inches}) \longrightarrow 6 \text{ feet } + 16 \text{ inches}\]

Now you have enough inches to subtract the 9 inches.

**Original Problem:**

- \(7 \text{ feet } 4 \text{ inches}\)
- \(-3 \text{ feet } 9 \text{ inches}\)

**This Example with Regrouping:**

- \(6 \text{ feet } 16 \text{ inches}\)
- \(-3 \text{ feet } 9 \text{ inches}\)
- \(3 \text{ feet } 7 \text{ inches}\)

**Check for Subtraction:**

- \(3 \text{ feet } 7 \text{ inches}\)
- \(+3 \text{ feet } 9 \text{ inches}\)
- \(6 \text{ feet } 16 \text{ inches}\)
- \(+1 \text{ foot } 12 \text{ inches}\)
- \(7 \text{ feet } 4 \text{ inches}\)

Answer: The difference between 7 feet 4 inches and 3 feet 9 inches is 3 feet 7 inches.

A “simpler” way to write this is as follows:

- **Step 1:** 4 inches at the top is too small to subtract 9 inches. We need to borrow more inches.

- **Step 2:** Borrow 1 foot from the 7 feet which leaves 6 feet.

- **Step 3:** Put the 1 foot into the conversion rule.

- **Step 4:** The 1 foot becomes 12 inches and is added to the 4 inches we already had.

- **Step 5:** Now there are 16 inches at the top so you can complete the subtraction to get a final answer. Remember labels!

Answer: The difference between 7 feet 4 inches and 3 feet 9 inches is 3 feet 7 inches.
Solve and check:

1) \(17 \text{ lbs } 5 \text{ oz}\)  
   \(- 9 \text{ lbs } 7 \text{ oz}\)

2) \(108 \text{ ft } 6 \text{ in}\)
   \(- 84 \text{ ft } 9 \text{ in}\)

Check:

3) \(12 \text{ hrs } 14 \text{ min}\)
   \(- 6 \text{ hrs } 39 \text{ min}\)

4) \(24 \text{ lbs } 8 \text{ oz}\)
   \(- 15 \text{ lbs } 14 \text{ oz}\)

Check:

5) \(12 \text{ ft } 2 \text{ in}\)
   \(- 7 \text{ ft } 11 \text{ in}\)

6) \(73 \text{ hrs } 42 \text{ min}\)
   \(- 11 \text{ hrs } 53 \text{ min}\)

Check:
**SUBTRACTION WITH BORROWING ACROSS ZEROS**

**Ex.A:** Find the difference between 5000 and 2783:

```
5 0 0 0
- 2 7 8 3
```

We can’t take 3 from 0 in the ones column, so we need to borrow a ten.

```
5 0 0 0
- 2 7 8 3
```

Looking at the tens column there is nothing there but a zero.

```
5 0 0 0
- 2 7 8 3
```

Draw a circle starting at the zero in the tens columns all the way to the 5, the first nonzero digit to the left.

```
4 9 9
5 0 0 0
- 2 7 8 3
```

The 5 in the thousands column equals 500 tens. We can borrow a ten from 500 tens, leaving 499 tens.

```
4 9 9 10
5 0 0 0
- 2 7 8 3
```

Take the extra ten and add it to the zero in the ones column and we have enough in the ones column to subtract. So subtract!

```
2 2 1 7
```

**NOW CHECK YOUR WORK!!!!**

Remember:  \[ \text{Bottom} + \text{Middle} = \text{Top} \]

<table>
<thead>
<tr>
<th>Bottom</th>
<th>2217</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Middle</td>
<td>+2783</td>
</tr>
<tr>
<td>Top</td>
<td>5000</td>
</tr>
</tbody>
</table>

Be sure this sum matches the top number given in the subtraction.

**Answer:** The difference between 5000 and 2783 is 2217.
MORE SUBTRACTION (with borrowing across zeros)

(Remember to check your answers!!)

Traditional Method:

1a) \[ 6007 - 1299 = 4708 \]
2a) \[ 40000 - 13562 = 26438 \]
3a) \[ 40080 - 27354 = 12726 \]
4a) \[ 40050 - 27384 = 12666 \]

Alternate Method(s), for example: Adding Up, Haitian Style, etc.

1b) \[ 6007 - 1299 = 4708 \]
2b) \[ 40000 - 13562 = 26438 \]
3b) \[ 40080 - 27354 = 12726 \]
4d) \[ 40050 - 27384 = 12666 \]

MISSING DIGIT SUBTRACTION:

\[ \begin{array}{c}
\phantom{0} \phantom{0} \phantom{0} 2 \phantom{0} \phantom{0} 3 \leftarrow \text{Top} \\
- \phantom{0} \phantom{0} \phantom{0} 4 \phantom{0} \phantom{0} 7 \phantom{0} \phantom{0} 5 \leftarrow \text{Middle} \\
\hline
\phantom{0} \phantom{0} \phantom{0} 2 \phantom{0} \phantom{0} 8 \phantom{0} \phantom{0} 6 \leftarrow \text{Bottom}
\end{array} \]

DON’T DO IT AS SUBTRACTION: Rewrite it as related addition, it’s much easier to solve:

Remember: \[ \text{Bottom} + \text{Middle} = \text{Top} \] Now solve it as addition…

\[ \begin{array}{c}
2 \phantom{0} \phantom{0} 8 \phantom{0} \phantom{0} 6 \leftarrow \text{(Bottom)} \\
+ \phantom{0} \phantom{0} \phantom{0} 4 \phantom{0} \phantom{0} \phantom{0} 7 \phantom{0} \phantom{0} 5 \leftarrow + \text{(Middle)} \\
\hline
\phantom{0} \phantom{0} \phantom{0} 2 \phantom{0} \phantom{0} \phantom{0} 3 \leftarrow \text{(Top)}
\end{array} \]

CHECK by substituting the answers into the boxes in the given problem and see if the problem is mathematically accurate. Do the subtraction that you were originally given! Are the differences the same?
Find the **Missing Digits**: **First Rewrite as Missing Digit Addition**

NOTE: \[ \text{Bottom} + \text{Middle} = \text{Top} \]

Don’t forget to check your work: Substitute answers into the **given problem** and see if the subtraction works!

1) \[
\begin{array}{c}
\phantom{4} \phantom{2} \\
\phantom{-} 2 \\
4 \\
\hline
4 \\
\phantom{4} 5
\end{array}
\]

2) \[
\begin{array}{c}
6 \\
\phantom{-} 2 \\
\phantom{-} 8 \\
\hline
2 \\
\phantom{2} 9 \\
\phantom{2} 5
\end{array}
\]

3) \[
\begin{array}{c}
\phantom{2} \phantom{2} \\
\phantom{-} 2 \\
\phantom{-} 7 \\
\hline
2 \\
\phantom{2} 8 \\
\phantom{2} 5
\end{array}
\]
Homework 1.6  Name: __________________________

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

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<tbody>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>2) Digits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Mixed Unit Problems</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 4 – 6, find the following differences and check your work:

<table>
<thead>
<tr>
<th>Question</th>
<th>Calculation</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>4)</td>
<td>837 - 591</td>
<td></td>
</tr>
<tr>
<td>5)</td>
<td>635 - 257</td>
<td></td>
</tr>
<tr>
<td>6)</td>
<td>4000 - 1184</td>
<td></td>
</tr>
</tbody>
</table>

For questions 7 – 9, estimate the following differences (show the numbers you are using):

<table>
<thead>
<tr>
<th>Question</th>
<th>Calculation</th>
<th>Numbers Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>7)</td>
<td>82 - 57</td>
<td>80, 60</td>
</tr>
<tr>
<td>8)</td>
<td>742 - 356</td>
<td>750, 360</td>
</tr>
<tr>
<td>9)</td>
<td>10,598 - 856</td>
<td>10,600, 850</td>
</tr>
</tbody>
</table>
Refer to the following chart to answer questions 10 – 11:

This chart shows the Median US Salaries for the years 1965 and 2012 as well as these salaries adjusted for inflation to the year 2014.

<table>
<thead>
<tr>
<th></th>
<th>In 1965</th>
<th>In 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual median salary in each year:</td>
<td>$6900</td>
<td>$51,017</td>
</tr>
<tr>
<td><strong>Median salary adjusted for inflation to the year 2014:</strong></td>
<td>$52,111</td>
<td>$52,862</td>
</tr>
</tbody>
</table>

10) Approximately how much more is the average worker making in 2012 as compared to 1965, based on the median salaries as adjusted for inflation to the year 2014?

11) What is the exact increase in the median adjusted salary from 1965 to the median adjusted salary in 2012?

For questions 12 – 14, find the following differences and check your work:

12) 12 weeks 4 days - 5 weeks 3 days
13) 14 feet 8 inches - 7 feet 11 inches
14) 8 hrs 29 min - 6 hrs 45 min
For questions 15 – 17, solve for the missing digits and check the differences:

NOTE: Remember, it is easier to solve these by first rewriting them as related addition.

15) \[ \begin{array}{c}
7 \boxed{} \\
- \boxed{} 8 \\
\hline
4 \boxed{} 5
\end{array} \]

16) \[ \begin{array}{c}
\boxed{} \boxed{} 5 \\
- 3 7 \boxed{} \\
\hline
4 \boxed{} 3 4
\end{array} \]

17) \[ \begin{array}{c}
\boxed{} 0, \boxed{} 0 \boxed{} \\
- 1 \boxed{}, 8 \boxed{} 2 \\
\hline
7, 1 4 8
\end{array} \]
Chapter 1.7

WORD PROBLEMS - PROBLEM SOLVING TECHNIQUES

1. Read the problem through twice.
   a. The first time you read it, just read. Take your time and **do not make any assumptions**.
   b. On the 2nd read through highlight or circle any information that is numerical whether or not it is a numeral or a word.
   c. Make a list of the numerical information and **LABEL IT**!! Organization is key. You don’t want to constantly go back and reread the problem to find out what a specific number represents.
   d. Draw a picture and label that if necessary.

2. Write out in words what you know or need to know.

3. Rewrite the sentence substituting numbers where you can. If you still don’t have a number call it “x”.

4. Solve for “x”.

5. Does your answer make sense? If yes, go to the next step, if not check your work.

6. Answer the question that was asked.
   A. Use a full sentence and give a reason for your answer.
   B. A simple yes or no is not sufficient. Explain the answer.
      i. “Yes” or “No” is not an answer.
      ii. “Yes, you have enough money to cash a check for $537, because the ending balance in your checking account is $788, which is enough money to cover the check” is an answer.
   C. A simple number is not sufficient. It must be followed with information to explain the number.
      i. For example, “34” is not an answer.
      ii. “The trip was 34 miles long” is an answer.
Car Odometers: The **odometer** on a car records the miles a car has been driven from the day it rolled out of the factory.

**Ex. A:** When a used car was purchased 5 years ago, it had 32,856 miles on the odometer. That means the car had been driven 32,856 miles by the first owner. The car now has 144,293 miles on the odometer. **How many miles** was the car driven by the second owner?

5yrs = the number of years the second owner had the car
32,856 mi = the number of miles on the car when it was purchased by the second owner
144,293 mi = the number of miles on the car today
x = the number of miles the second owner has driven the car since they bought it

\[
\begin{align*}
\text{Miles driven by 1}^{\text{st}} \text{ owner} + \text{miles driven by 2}^{\text{nd}} \text{ owner} &= \text{current miles on the car today} \\
32,856 + x &= 144,293 \\
-32,856 &= -32,856 \\
x &= 111,437
\end{align*}
\]

The answer makes sense because the second owner put a lot of miles on the car! So, the final step is to present the answer in a sentence.

**Answer:** The second owner drove the car 111,437 miles (over the last 5 years).

**Perimeter Problems:**

Definition: **Perimeter:** – The distance around the outside **edges** of any figure or shape.

To find the perimeter, find the **sum** of all the sides:

Side 1 + Side 2 + Side 3 + ...any other remaining sides = the perimeter of the shape

**Ex. B:** Find the perimeter of a rectangle with length equal to 43ft and width equal to 17ft.

\[
P = \text{Sum of all sides} \\
P = 43 \text{ ft} + 17 \text{ ft} + 43 \text{ ft} + 17 \text{ ft} \\
P = 120 \text{ feet}
\]

**Answer:** The perimeter of the rectangle is 120 feet.
Checking Accounts:

Your bank keeps a current account of all your transactions.

**Deposits** are added to your current balance and **checks** are subtracted from your current balance. Your bank does this once per day.

All the deposits you make in a day are summed and then added to your balance. Then all the checks/debits that same day are summed and subtracted from your balance. If the total amount for the checks/debits is greater than your balance they will bounce **all** of your checks and charge you a $35 to $50 fee for **each check/debit** you bounce.

**For example** if your balance is $320 and you withdraw money and write 3 checks totaling $321, the bank will bounce everything and charge you 3 “overdrawn” fees, totaling anywhere from $105 to $150. Therefore, knowing your current balance is extremely important.

How do you keep track? **Write everything down.**

**Ex. C:** Suppose your current checking balance is $1036, and you make two deposits of $450 and $187. You also write checks that day for $85, $426, and $518. Can you write another check (or use your debit card) for those shoes you really, really want that cost $288?

**Sum the Deposits:** (Add to balance)

$1036 = starting balance

$450 + $187 = $637 are the total deposits

$1036 + $637 = $1673 is the balance after the deposits

**Sum the checks:** (Subtract from balance)

$85 + $426 + $518 = $1029 are the total checks

$1673 – $1029 = $644

Answer: You can buy the shoes since you have $644 in your checking account.

**NOTE:** One last question, you can afford those shoes right now but do you have other expenses in the coming weeks that you **have** to cover like transportation to and from school, books, new brakes for your car, rent, food, OMG, think ahead.
**PROBLEM SOLVING SUMMARIZED**

1. **READ**
   Understand what is given and what you are looking for. (Underline or circle.)
   Is there any *unnecessary* information provided? (Cross out.)

2. **PLAN**
   Organize the given information. Make a list of each number and what it means.
   Draw a picture if needed. Label the picture. Write any formulas you may need.
   Plan your strategy. Stop thinking of math and think of real life.

3. **SOLVE**
   Be sure to include any labels to give the answer meaning! (Write a sentence.)

4. **CHECK**
   a) Does the answer make sense??
   b) Check the arithmetic.

1) At NCC’s theater, ticket prices were $9 for adults, $6 for students 13 to 19 years of age, and $4 for children 12 years old and younger. Nicolas is 13 years old and his sister Lauren is 9. If their parents take them to the theater, how much will the four of them pay?

2) Find the perimeter of the following figure:

```
7 feet
11 feet
13 feet
```

```
11 feet
```

Mathematical Foundations by Debra J. Grodenchik, Ph.D., Jennifer W. Kohut, et al at Nassau Community College is licensed under [CC-BY-SA 4.0](https://creativecommons.org/licenses/by-sa/4.0)
3) The sketch below shows a map of the eastern United States:

a) How far is it from Buffalo to Charlotte through Baltimore?  
b) How much shorter is it to go from Buffalo directly to Charlotte (without going through Baltimore)?

![Map of the eastern United States showing distances between Buffalo, Charlotte, and Baltimore.]

4) The height of Mount Marcy, the highest point in New York State, is five thousand, three hundred forty-three feet. The height of Mount Washington, the highest point in New Hampshire, is six thousand, two hundred eighty-eight feet. How much higher is Mount Washington than Mount Marcy?

Notice that there are four different ways to write this answer:

Mount Washington is 945 ft. higher than Mount Marcy.  
Mount Marcy is 945 ft. lower than Mount Washington.  
Mount Washington is higher that Mount Marcy.  The difference in their heights is 945 ft.  
Mount Marcy is lower than Mount Washington.  The difference in their heights is 8,709 ft.
PICTURING THE DIFFERENCE

Look for the difference phrase: “…(number) more than…” or “…(number) less than…” or “…how much more than…” or “…how much less than…”

The number directly to the left of the “more than” or “less than” wording in the sentence represents a difference. Not only is a difference the answer to subtraction, but a difference can also represent the distance between two numbers.

These sentences can be worded many different ways, as either greater or less than ideas. Also, the difference can be stated in the middle of the sentence or at the end (as an after-thought).

For example:  I am two inches taller than my sister.
My sister is two inches shorter than I am.
Rephrased: I am taller than my sister. (The difference in our heights is two inches.)
My sister is shorter than I am. (The difference is our heights is two inches.)

For example: It is ten degrees cooler today than it was yesterday.
It was ten degrees warmer yesterday than it was today.
Rephrased: It is cooler today than yesterday (ten degrees cooler).
It was warmer yesterday than it is today (ten degrees warmer).

Ex.A: 37 is **56** less than what number? Knowing the difference = 56, we can rephrase the rest of this problem as “37 is less than what number?” (Here we are looking for the bigger #.)

On a number line: 37 is on the left and the missing number is on the right. 56 is in the middle.

```
37  ??  56

Since this is a picture of number line addition
we find the missing number by adding to get 37 + 56 = 93
The missing number = 93.
```

Ex.B: 56 is **37** more than what number? Knowing the difference = 37, we can rephrase the rest of this problem as “56 is more than what number?” (Here we are looking for the smaller #.)

On a number line: 56 is on the right and the missing number is on the left. 37 is in the middle.

```
??  37  56

Since this is a picture of number line subtraction
we find the smaller (missing) number by subtracting
to get 56 – 37 = 19. The missing number = 19.
```
NOTE: When a problem says “…how much more than…” or “…how much less than…” the number that represents the difference is missing. Because the difference is the answer to subtraction, in these problems you simply need to subtract the two numbers to solve the problem. For example: How much more money will I make after receiving a raise in pay? This is asking one to find the difference in their salaries before and after a raise in pay is received.

Ex.C: 85 is how much more than 37?

Another way to phrase this problem is “85 is more than 37, but how much more?”

---

This is a picture of number line subtraction where you are looking for the distance between 2 numbers on the number line.

But, because we are simply looking for a difference, we can just subtract 85 – 37 = 48 to get the difference (the missing number): The missing number is 48, so 85 is 48 more than 37.

**Picturing the difference – practice**

In purely mathematical problems it sometimes seems, from the language used, that you need to add when you actually need to subtract (or vice versa) in order to solve a problem. Be careful!!

Use the ideas above to solve the following problems:

1. 45 is 56 less than what number?
2. What number is 56 less than 82?
3. 45 is how much less than 82?
4. 156 is 45 more than what number?
5. What number is 56 more than 145?
6. 146 is how much more than 54?
WAITING ON LINE AT THE BOOKSTORE

It is the first week of classes for the semester. You want to get your books as soon as possible to beat the rush. You get to the bookstore at 11:00 am and see a line has formed outside, so you ask the guard on duty how long you can expect to wait.

She provides you with the following information:
- She must wait until 6 people exit before she may let 6 people enter the bookstore.
- Student are leaving at the rate of 2 students per minute.
- She just let a group of 6 students into the store.
- Students who are exiting tell you they spent an average of 15 minutes gathering books and supplies and another 10 minutes waiting on line to check out.

There are currently 38 people in line ahead of you. You know it takes ten minutes to walk to your 12:00 class. Can you buy your books and still make it to your noon class on time?

Note: This is a difficult problem. Go back to page 67 and 70 and use the problem solving methods there to help solve this problem.
Before you do your homework please review the terms in this chapter and the method(s) for solving word problems. If you are not familiar with any or all of these methods: Refer to your notes OR see a tutor OR go to Open Time in lab.

Use any appropriate method(s) that will help you arrive at reasonable solutions for the following word problems. Remember to show all work & label all answers.

1) Professor A. teaches three classes. There are 42 students in her MAT 101 class, 32 students in her MAT 001 class, and 38 students in her NCC 101 class. No student is enrolled in more than one class with Professor A. How many students’ names does she need to learn for the semester?

2) Some jobs pay one rate for the first 40 hours you work in a single week. When an employee works more than 40 hours in a particular week (overtime hours) they are paid at a higher rate for those extra hours worked. If you work 54 hours in a single week, how many overtime hours did you work that week?
3) Professor A’s checking account balance on her last bank statement was $348. Since then she deposited two paychecks, where each paycheck was $1546. She wrote a check for her daughter’s monthly tuition plan for $2200, a check for $235 for a pair of orthopedic shoes, and a check for $187 to pay last month’s LIPA bill. What is the balance of the account after all of these transactions have been completed?

4) If a class starts at 2:46 pm and ends at 4:18 pm, how long is the class?

5) I need to put a frame around a square picture. Each side of the frame needs to be 9 inches long. How much framing should I buy?
6) My niece was 3 ft. 11 in. tall and then grew 8 inches last year. My aunt, on the other hand, was 5 ft. 6 in. and she shrank 4 inches last year. Who was taller and by how much at the end of last year?

7) A train was supposed to leave at 6:48am but was delayed 37 minutes. If the train travels for 3 hrs 48 min, what time will the train arrive at its destination?
8) *The Blind Side* by Michael Lewis has 339 pages. I read the first two chapters last night which brought me to page 44. Chapter three has 30 pages and chapter four has 28 pages. I plan on reading chapters three and four tonight. If I do that, how many pages will I have left to read in order to finish the book?

9) My son is 17 years 7 months old and my daughter is 25 years 3 months old. How much older is my daughter?
**Associative Law of Addition**

*Definition:* Regrouping more than 2 numbers results in identical sums. (Number order remains the same.)

*Example:* \((1 + 2) + 3 = 1 + (2 + 3)\), regrouping numbers will not change the final sum.

**Commutative Law of Addition**

*Definition:* The sum of 2 numbers is the same regardless of number order.

*Example:* \(3 + 5 = 5 + 3\), switching (commuting) the order of the numbers doesn't change the sum.

**Difference**

*Definition:* 1) The answer to a subtraction problem.

*Example:* In the sentence \(10 - 6 = 4\), 4 is the difference between 10 and 6.

*Definition:* 2) The distance between two numbers on the number line.

*Example:* There are 4 spaces between 6 and 10 on the number line.

**Digits**

*Definition:* The first ten whole numbers.

*Example:* 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are the digits.

**Equal to (=) / Equation**

*Definition:* An equation states that two expressions are equal. Two expressions are equal if they have the same value.

*Example:* \(8 = x + 5\) is an equation. It states that 8 has the same value as \(x+5\).

**Expanded Notation**

*Definition:* The sum of the values of the NON-ZERO digits in a number.

*Example:* 2,000,305,076 in expanded notation is 2,000,000,000+300,000+5,000+70+6.

**Greater than (>)**

*Definition:* A number is greater than another number if it appears to the right of the other number on the number line.

*Example:* \(15 > 2\), because 15 is to the right of 2 on the number line.

**Group Names**

*Definition:* Numbers are arranged in 3-digit groups and each group is named.

*Example:* In 200,736 thousands is the group name for the 200 and there is no stated group name for the 736 (its group name is units).

*Example:* Trillions, Billions, Millions, Thousands, Units, these are the first five group names.

**Identity Property of Addition**

*Definition:* Adding zero to a number gives the same number as the sum.

*Example:* \(18 + 0 = 18\), adding 0 to a number adds no value to the number, (18 kept its “identity”).

**Less Than (<)**

*Definition:* A number is less than another number if it appears to the left of the other number on the number line.

*Example:* \(4 < 9\), because 4 is to the left of 9 on the number line.

**Natural or Counting Numbers**

*Definition:* The numbers that start with 1 and continue forever.

*Example:* 1,2,3,4,5,6,7,8,9,10,11...are the Natural Numbers.

**Perimeter (sum of all sides)**

*Definition:* The sum of the lengths of all the sides of any shape.

*Example:* A rectangle whose length is 5 in. and width is 3 in. has a perimeter of \(5 + 3 + 5 + 3 = 16\) in.

**Standard Notation**

*Definition:* A number written using digits.

*Example:* 2,000,305,076 is a number written in standard notation.
Whole numbers
Definition: The Natural Numbers with zero added at the beginning of the list.
Example: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,...are the Whole Numbers.

Word Names
Definition: A number written using words.
Example: A word name for 2,000,305,076 is two billion, three hundred and five thousand, seventy-six.

IMPORTANT IDEAS AND CONCEPTS YOU NEED TO KNOW

Counting Triangles: Determining the total number of triangles in a given shape that can be made from the individual pieces and all possible combinations of the pieces that have common sides.
See pages 29 - 33.

Estimate: To find an approximate answer to any mathematical problem by first rounding all numbers and then performing the indicated operation with the rounded numbers.
See page 39 and 42.

Missing Digit Addition and Subtraction: Determine the digits necessary to make a mathematical problem correct.
See pages 47 - 48 and 60.

Mixed Unit Addition and Subtraction: A method for adding and subtracting in problems with different units of measurement.
For Example: feet and inches, months and years, minutes and hours, pounds and ounces, etc...
See pages 26 - 27 and 57.

Place Value: Each non-zero digit has a value that depends on its place in the number.
For Example: The value of the digit 5 in 537 is 500 and its value in 375 is 5. (Think money!)
See page 4 - 5.

Related Addition and Subtraction Sentences: Using a given addition sentence there are two related subtraction sentences, both starting with the sum.
For Example: From the addition sentence 8 + 7 = 15 we can write 15 – 8 = 7 and 15 – 7 = 8.
See page 45.

Rounding: Determining an approximate value of a number based on its proximity to numbers above and below it in a specific place value.
For Example: 752 is closest to 750 when counting by tens.
See page 39 - 40.
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) What is the smallest 6-digit number?</td>
<td>100000</td>
</tr>
<tr>
<td>2) What is the largest 3-digit number?</td>
<td>999</td>
</tr>
</tbody>
</table>

For quest. 3 – 7, use this number line:

| 1 | 2 | a | c | 5 | d | 7 | b | 9 | 10 |

<table>
<thead>
<tr>
<th>For questions 3 – 4, find the following values:</th>
<th>For questions 5 – 7, insert the correct symbol &lt;, &gt;, = between the following pairs of numbers:</th>
</tr>
</thead>
<tbody>
<tr>
<td>3) a =</td>
<td>4) c =</td>
</tr>
<tr>
<td>5) b d</td>
<td>6) c d</td>
</tr>
<tr>
<td>7) c 9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For questions 8 – 10, write in expanded notation:</th>
<th>For questions 11 – 13, write in standard notation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>8) 23,103,072</td>
<td>11) 40,000,000 + 5,000,000 + 50,000 + 800 + 5</td>
</tr>
<tr>
<td>9) 17,005,000,204</td>
<td>12) 70,000,000,000 + 4,000,000,000 + 800,000,000 + 20,000,000 + 70 + 5</td>
</tr>
<tr>
<td>10) 920,000,000,045,022</td>
<td>13) 900,000,000,000,000 + 2,000,000 + 80,000 + 3000 + 500 + 60 + 7</td>
</tr>
</tbody>
</table>
For questions 1–3, write a **word name** for each of the following numbers:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>378,029,400</td>
</tr>
<tr>
<td>2</td>
<td>48,000,004,073</td>
</tr>
<tr>
<td>3</td>
<td>19,350,000,000,018</td>
</tr>
</tbody>
</table>

For questions 4–6, write the following **word names** in **standard notation**:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Forty-nine billion, eight hundred fifty-six million, two hundred thirty-five thousand, one hundred nineteen.</td>
</tr>
<tr>
<td>5</td>
<td>Three billion, two million, fourteen thousand.</td>
</tr>
<tr>
<td>6</td>
<td>Twenty-three trillion, forty-five million.</td>
</tr>
</tbody>
</table>

For questions 7–9, give the place value of the **digit 3** in the following:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>36,052</td>
</tr>
<tr>
<td>8</td>
<td>413,206,759</td>
</tr>
<tr>
<td>9</td>
<td>6,320,498,526,924</td>
</tr>
</tbody>
</table>

For ques. 10–13, give the digit that represents the places in the number: 135,792,468,263,845

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Ten-thousands</td>
</tr>
<tr>
<td>11</td>
<td>Billions</td>
</tr>
<tr>
<td>12</td>
<td>Hundreds</td>
</tr>
<tr>
<td>13</td>
<td>Hundred-trillions</td>
</tr>
</tbody>
</table>
1) If you apply the commutative law of addition to 36 + 42, what would it look like and will it change the answer?

2) If I ask you to add 4, 5, and 6, what two numbers would you add first? Does it matter?

3) Find the sum of 2098 and 562.

For questions 4 – 6, find the following sums:

<table>
<thead>
<tr>
<th>4)</th>
<th>5)</th>
<th>6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>11 + 8 + 2 + 29 = 40</td>
<td>814</td>
</tr>
<tr>
<td>81</td>
<td></td>
<td>233</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td>562</td>
</tr>
<tr>
<td>+ 73</td>
<td></td>
<td>+ 401</td>
</tr>
</tbody>
</table>
For questions 7 – 10, find the sum for each of the following mixed unit problems:

7)  
7 years 5 months  
+ 4 years 6 months

8)  
3 feet 4 inches  
+ 5 feet 9 inches

9)  
7 hours 35 minutes  
+ 6 hours 25 minutes

10)  
2 pounds (lbs) 13 ounces (oz)  
+ 4 pounds (lbs) 7 ounces (oz)

For questions 11 – 12, figure out how many triangles are in each picture:

11)  

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12)  

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For questions 1 – 3 round the given numbers to the following places (or round to the nearest):

<table>
<thead>
<tr>
<th></th>
<th>a) 10’s place</th>
<th>b) 100’s place</th>
<th>c) 1000’s place</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>985</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2)</td>
<td>1,438</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3)</td>
<td>27,795</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 4 – 6, Estimate the sum for each of the following problems by rounding each number to the indicated place value: (Please show all the rounded numbers you are using.)

<table>
<thead>
<tr>
<th>tens place</th>
<th>hundreds place</th>
<th>thousands place</th>
</tr>
</thead>
<tbody>
<tr>
<td>4) 815</td>
<td>5) 635</td>
<td>6) 5,213</td>
</tr>
<tr>
<td>78</td>
<td>24</td>
<td>978</td>
</tr>
<tr>
<td>11</td>
<td>985</td>
<td>24</td>
</tr>
<tr>
<td>563</td>
<td>342</td>
<td>14,838</td>
</tr>
<tr>
<td>+ 59</td>
<td>+ 814</td>
<td>+ 2,782</td>
</tr>
</tbody>
</table>

7) Estimate the sum of:

- 12,825
- 8,915
- 356
- + 34,204

8) In 2014 the national debt had climbed to $14,365,872,483,905. To the nearest trillions place, how much is the country in debt?
For questions 1 – 3, solve for \( x \) and check your work:

1) \( x + 48 = 79 \)

check:

2) \( x - 27 = 45 \)

check:

3) \( 342 + x = 921 \)

check:

For questions 4 – 6, find sum for each of the following mixed unit problems:

4) 5 weeks 2 days
6 weeks 4 days
+ 3 weeks 5 days

5) 7 hrs. 42 min.
2 hrs. 38 min.
+ 3 hrs. 51 min.

6) 7 years 9 months
2 years 8 months
+ 3 years 11 months

For questions 7 – 9, solve for the missing digits and check the sums:

7)  

\[
\begin{array}{c}
6 \ \square \\
+ \quad \square \ 5 \\
\hline \\
\square \ 2 \ \ 3
\end{array}
\]

8)  

\[
\begin{array}{c}
\square \ 9 \ 6 \\
\hline \\
\square \ 3 \ \square \ 8 \\
+ \quad 7 \ 9 \ \square \\
\hline \\
\square \ 5 \ \square \ 0
\end{array}
\]

9)  

\[
\begin{array}{c}
5 \ 8 \ 2 \ \square \\
\hline \\
2 \ \square \ 9 \ 4 \\
+ \quad 7 \ 1 \ \square \ 6 \\
\hline \\
1 \ \square , 5 \ 7 \ 5
\end{array}
\]
For questions 1 – 6, find the following differences and check your work:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>918 - 473</td>
<td></td>
</tr>
<tr>
<td>2)</td>
<td>1085 - 598</td>
<td></td>
</tr>
<tr>
<td>3)</td>
<td>60805 - 42139</td>
<td></td>
</tr>
<tr>
<td>4)</td>
<td>18 wks 6 days - 11 wks 5 days</td>
<td></td>
</tr>
<tr>
<td>5)</td>
<td>6 ft. 5 in. - 3 ft. 11 in.</td>
<td></td>
</tr>
<tr>
<td>6)</td>
<td>10 hrs. 33 min. - 6 hrs. 48 min.</td>
<td></td>
</tr>
</tbody>
</table>

Check:   

Check:   

Check:   

Check:   

Check:   

Check:
For questions 7 – 9, estimate the following differences:

7)  
\[
\begin{array}{c}
48,392 \\
-21,897 \\
\end{array}
\]

8)  
\[
\begin{array}{c}
4398 \\
-752 \\
\end{array}
\]

9)  
\[
\begin{array}{c}
37,912 \\
-298 \\
\end{array}
\]

For questions 10 – 12, solve for the missing digits and check the differences:

10)  
\[
\begin{array}{c}
8 \_ \_ \\
- \_ 7 \\
\hline
4 & 5
\end{array}
\]

11)  
\[
\begin{array}{c}
\_ \_ 4 \\
- 47 \_ \\
\hline
4 & 3 & 5
\end{array}
\]

12)  
\[
\begin{array}{c}
\_ \_ 4, 32 \_ \\
- 1, 35 \_ \\
\hline
4 & 1, 96 \_ 
\end{array}
\]
1. A family drove from New Jersey to New Orleans. It was a horrible trip, no air conditioning, the mom smoked in the car and the little brother got sick outside of Nashville, Tennessee. On the first day they drove 388 mi, on the next day they drove 375 mi, on the third day they only drove 256 mi. The last day of the trip they drove 312 mi to finally get to New Orleans. How many miles was the entire trip from New Jersey to New Orleans?

2. Professor A. works hard for her money. She gets paid every other week. Her gross paycheck of $1987 looks really good at first, however, there are a few deductions. The federal government takes $556, the governor of NY and FICA each take $139, and Social Security wants its share of $89. How much is her net* paycheck after all of these deductions?

*Net pay means how much money one takes home after all deductions are subtracted from the gross pay (the total pay before any deductions are taken out).
3. The distance from Garden City, NY to Buffalo, NY is 403 miles. Binghamton, NY lies on the road between the two and is 205 miles from Garden City. What is the distance between Binghamton and Buffalo?

4. Grandma was 5 ft. 1 in. tall and shrank 4 inches in one year. Her grandson Jack was 4 ft. 6 in. and grew 7 inches that same year. Who was taller at the end of the year and how much taller was he/she?
5. After the tornados of September 16, 2010 many people found it took them hours longer to get home from Manhattan because of all the problems on the roads and the trains. My sister left work in Manhattan at 5:08 pm and, after two subway and two bus rides later, finally made it home by 11:30 that night. It normally takes her 40 minutes to get home. How much longer did it take her to get home that day as compared to any ordinary day?

6. I want to make a frame for a picture using the 14 ft. of framing material I have left from another project. If I want the frame to be 3 ft. wide, what is the longest the picture can be so I can use all of this framing material?
1.1 extra practice answers:

1) 100,000  
2) 999  
3) 3  
4) 4  
5) >  
6) <  
7) <  
8) 20,000,000 + 3,000,000 + 100,000 + 3000 + 70 + 2  
9) 10,000,000,000 + 7,000,000,000 + 5,000,000 + 200 + 4  
10) 900,000,000,000 + 20,000,000,000 + 40,000 + 5,000 + 20 + 2  
11) 45,050,805  
12) 74,820,000,075  
13) 900,000,002,083,567

1.2 extra practice answers:

1) Three hundred seventy eight million, twenty nine thousand, four hundred  
2) Forty eight billion, four thousand, seventy three  
3) Nineteen trillion, three hundred fifty billion, eighteen  
4) 49,856,235,119  
5) 3,002,014,000  
6) 23,000,045,000,000  
7) ten-thousands  
8) (one) millions  
9) hundred-billions  
10) 6  
11) 2  
12) 8  
13) 1

1.3 extra practice answers:

1) 42 + 36; no  
2) *  
3) 2660  
4) 203  
5) 50  
6) 2010  
7) 11 yrs 11 mo  
8) 9 ft 1 in  
9) 14 hrs  
10) 7 lbs 4 oz  
11) 5 triangles (see below for more detail)  
12) 8 triangles (see below for more detail)

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bit</td>
<td>a d</td>
<td>2</td>
</tr>
<tr>
<td>2 bit</td>
<td>ab</td>
<td>1</td>
</tr>
<tr>
<td>3 bit</td>
<td>abc</td>
<td>1</td>
</tr>
<tr>
<td>4 bit</td>
<td>abcd</td>
<td>1</td>
</tr>
</tbody>
</table>

| Total = 5 triangles |

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bit</td>
<td>a b c d</td>
<td>4</td>
</tr>
<tr>
<td>2 bit</td>
<td>ab bc cd ad</td>
<td>4</td>
</tr>
<tr>
<td>3 bit</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>4 bit</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

| Total = 8 triangles |

* These Questions & Answers may require some discussion.
1.4 extra practice answers:
1a) 990  b) 1000  c) 1000  
2a) 1440  b) 1400  c) 1000  
3a) 27,800  b) 27,800  c) 28,000  
4) 1530  
5) 2700  
6) 24,000  
7) *  
8) $14,000,000,000,000  

1.5 extra practice answers:
1) x = 31  
2) x = 72  
3) x = 579  
4) 15 weeks 4 days  
5) 14 hrs 11 min  
6) 14 years 4 months  
7) hundreds = 1, tens = 5, ones = 8  
8) thousands = 1, hundreds = 3, tens = 5, ones = 6  
9) thousands = 5, hundreds = 5, tens = 5, ones = 5  

1.6 extra practice answers:
1) 445  
2) 487  
3) 18,666  
4) 7 weeks 1 days  
5) 2 ft 6 in  
6) 3 hr 45 min  
7) 30,000  
8) *  
9) *  
10) tens = 3, ones = 2  
11) hundreds = 9, tens = 1, ones = 9  
12) ten-thousands = 5, thousands = 2, hundreds = 9, tens = 2, ones = 1,  

1.7 extra practice answers:
1) 1331 miles  
2) $1064  
3) 198 miles  
4) Jack was 4 inches taller  
5) 5 hours 42 minutes  
6) 4 feet  

* These Questions & Answers may require some discussion.
CHAPTER 2

PROBLEM SOLVING

WITH

MULTIPLICATION AND DIVISION

OF

WHOLE NUMBERS
Chapter 2.1  MULTIPLICATION CONCEPTS AND PROPERTIES

Multiplication is a quicker form of adding the same number over and over again.

For example: Find the following sums:

Ex. A: \( 2 + 4 + 1 + 8 + 4 = \)

\[
(2 + 4) + 1 + 8 + 4 = \\
(6 + 1) + 8 + 4 = \\
(7 + 8) + 4 = \\
15 + 4 = 19 \text{ is the sum.}
\]

Ex. B(i): \( 4 + 4 + 4 + 4 + 4 + 4 = \)

Since we are adding four “six times” it can be written as “six times four” or \( 6 \times 4 \).

Ex. B(ii): \( 4 + 4 + 4 + 4 + 4 + 4 = 6 \times 4 = 24 \) is the product of 6 and 4.

Definition: **Product**: The result of (or answer to) a multiplication problem.

We can multiply across from left to right OR we can multiply vertically from top to bottom

\[
3 \times 4 = 12 \\
\]

Multiplication can be visualized in an array:

```
* * * * 
* * * * 
* * * * 
```

This can be seen as either:

3 rows of 4 stars in each row OR 4 columns of 3 stars in each column

3 rows x 4 stars in each row OR 4 columns x 3 stars in each column

NOTE: Know your basic multiplication facts found on page 466 in the back of this text. Speed and accuracy are important.

There are many ways to indicate multiplication. The \( \times \), \( * \), \( \cdot \), ( ) all indicate multiplication. When a number is written next to a variable (an unknown value), then it means multiply.

Ex. C: Write four times three with different notation:

a) \( 4 \times 3 \)  b) \( 4 \ast 3 \)  c) \( 4 \cdot 3 \)  d) \( (4)(3) \)

e) \( 4(3) \)  f) \( (4)3 \)  g) \( 3y \), where \( y = 4 \)
PROPERTIES OF MULTIPLICATION

Commutative Law: \[ A 	imes B = B 	imes A \]
\[ 3 \times 4 = 4 \times 3 \]

Associative Law: \[ (A \times B) \times C = A \times (B \times C) \]
\[ (3 \times 4) \times 5 = 3 \times (4 \times 5) \]

Identity Property: \[ A \times 1 = A \]
\[ 3 \times 1 = 3 \]

Zero Property: \[ 0 \times \text{ANYTHING} = 0 \]
\[ \text{and ANYTHING} \times 0 = 0 \]

FACTORS OF A NUMBER

Definition: Factors: The numbers being multiplied. Factor \times Factor = Product

Factors are also known as divisors (coming up in a later section in this chapter!)

NOTE: To find FACTORS/DIVISORS of a number DIVIDE the number up into smaller pieces! The given number is divided by the Natural Numbers (in order). If they “go in evenly” (no remainder or decimals result), the pairs of numbers (the number you divided by and the corresponding answer to that division) are factors/divisors.

Ex. A: Find all the factors of 12. Remember: the identity property \((1 \times N = N)\) says that 1 and the number itself are always factors, so, start with 1 & the number itself.

Method A: This method uses knowledge of basic multiplication facts and lists pairs of factors.

1
2
6
3
4

\[ \times \] Stop!

4 is a repeated number, you can stop!

NOTE: As long as you look for factors in natural number order, you are done looking when you repeat a number on the list.

Show the pairs of factors when answering in a sentence: The factors of 12 are 1, 2, 3, 12, 6, 4.

Both of the following methods use division to find pairs of factors. Remember to divide the given number by natural numbers, in order, to find factors.

You are done when you repeat a number on the list.

Method B

\[
\begin{array}{c|c|c}
12 & 12 & 12 + = \\
1 & 12 & 12 + 1 = 12 \\
2 & 6 & 12 + 2 = 6 \\
3 & 4 & 12 + 3 = 4 \\
4 & done! & 12 + 4 done! \\
\end{array}
\]

NOTE: The factors of 12 are easy to “see” if you know your times tables (See Appendix A).

Answer: The factors of 12 are 1, 2, 3, 4, 6, and 12.
What if the number has hidden factors that are not in the times table or your multiplication facts are okay but not stellar? Methods B and C shown on the previous page will work like a charm.

NOTE: The 3 previous methods can be done with a four function (+, -, x, and ÷) calculator.

Ex. B: Find all the factors of 42.

Method A:

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 6 & 7 & 14 & 21 \\
42 & & & & & & \end{array}
\]

Stop!

Answer: The factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

Method B

\[
\begin{array}{c|c}
42 & \div \equiv \\
1 & 42 \\
2 & 21 \\
3 & 14 \\
4 & 10.5 \\
5 & 8.4 \\
6 & 7 \\
7 & \text{done!}
\end{array}
\]

NOTE: The numbers increase as you go down the column and the “gap” between the numbers decreases.

Answer: The factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

Find all the factors of the following:

1) List all factors of 36:

2) List all factors of 48:

3) List all factors of 51:

4) List all factors of 165:
If you are not familiar with any or all of the following terms or problems refer to your **notes** OR **Glossary/Important Ideas and Concepts** OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
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</thead>
<tbody>
<tr>
<td>1) Product</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Commutative Property of Multiplication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Associative Property of Multiplication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Zero Property of Multiplication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Identity Property of Multiplication</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7) What happens when you multiply any number by 1?

8) What two numbers are guaranteed to be factors of any number?

<table>
<thead>
<tr>
<th>For questions 9 – 10, identify the factors and product:</th>
</tr>
</thead>
<tbody>
<tr>
<td>9) 42 x 13 = 546</td>
</tr>
<tr>
<td>10) 71</td>
</tr>
<tr>
<td>x 6</td>
</tr>
<tr>
<td>426</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For questions 11 – 13, find all the factors of the following numbers:</th>
</tr>
</thead>
<tbody>
<tr>
<td>11) 15:</td>
</tr>
<tr>
<td>12) 18:</td>
</tr>
<tr>
<td>13) 25:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For questions 14 – 16, identify the following laws (Caution! No assumptions!):</th>
</tr>
</thead>
<tbody>
<tr>
<td>14) 5 x 3 = 3 x 5</td>
</tr>
<tr>
<td>15) 0 x 18 = 18 x 0</td>
</tr>
</tbody>
</table>

16) Complete the statement using the Associative Law:

6 x (7 x 8) = ________________
**MORE MULTIPLICATION:**

Distributive Law: \( A \times (B + C) = (A \times B) + (A \times C) \)

This law combines multiplication with addition (or subtraction).

**Ex A:**

\[
7 \times (5 + 8) = (7 \times 5) + (7 \times 8)
\]

Distribute the 7 outside the ( ) to the 5 and 8 inside the ( )

OR

Add in the ( ) first.

\[
7 \times 13 = 35 + 56
\]

Then multiply.

\[
91 = 91
\]

NOTE: Either way, you get the same answer.

**Ex. B:**

\[
\frac{(9 \times 8) + (9 \times 3)}{72 + 27} = 9 \times \frac{11}{9} = 99
\]

**Ex. C:**

\[
\frac{(9 \times 8) - (9 \times 3)}{72 - 27} = 9 \times \frac{5}{45} = 45
\]

Why would you need the Distributive Law? Mostly you’ve seen it in algebra as a way to simplify an algebraic expression.

**Ex. D:** \(2(x + 4)\) simplifies to \((2 \cdot x) + (2 \cdot 4) = 2x + 8\)

NOTE: Combining the Distributive Law and Expanded Notation can help make multiplication easier.

**Ex. E(i):** Find the product of 4(72) using Expanded Notation for 72 = 70 + 2.

Substituting the Expanded Notation for 72, the problem is now:

\[
4(70 + 2) = (4 \cdot 70) + (4 \cdot 2)
\]

\[
4(70 + 2) = 280 + 8
\]

\[
4(70 + 2) = 288
\]

Answer: The product of 4 and 72 is 288.
A more recognizable method:

**Ex. E(ii): (cont.)**

\[ \begin{array}{c}
72 \\
\times 4 \\
\hline
8
\end{array} \]

Start with the ones column, multiply the 4 x 2, and write the 8 in the ones column below the line.

Multiply the 4 x 7 and write your answer in the tens column. With basic multiplication and NO calculator, remember why place holders are used!! It’s not really 4 x 7 but 4 x 70 = 280 = 28 tens. The answer (28) is written starting from the tens column!

**Ex. F:**

Find the product of 34 and 53.

\[ \begin{array}{c}
53 \\
\times 34 \\
\hline
2 \\
n 1
\end{array} \]

Start with the ones column, multiply 4 x 3=12. 12 is 1 ten and 2 ones, write the 2 in the ones column and “carry” the 1 ten to the tens column.

Now multiply 4 x 5 and **add on the extra ten:**

4 x 5 = 20 + 1 = 21, and write your answer next to the 2. (Once you have added on the extra ten cross it out!)

With basic multiplication and NO calculator, remember why place holders are used!! Since the 3 is in the tens column we are multiplying by 30 not 3. So put a zero in the ones column as a place holder.

Multiply 3 x 3, and write the 9 next to the 0 place holder.

Multiply 3 x 5, and write the 15 next to the 9.

Add 212 + 1590 = 1802 is your answer.

Answer: **1802** is the **product** of 53 and 34.
Find the following products:

1) \( 71 \times 28 \)
2) \( 127 \times 45 \)
3) \( 243 \times 352 \)

**MULTIPLICATION WITH FINAL (ENDING) ZEROS**

**Ex. A:** What is 10 times 3?  
\( 10 \times 3 = 30 \)  
(It’s just a 3 with a zero.)

**Ex. B:** What is 100 times 3?  
\( 100 \times 3 = 300 \)  
(It’s just a 3 with 2 zeros.)

**Ex. C:** What is 10 x 10?  
\( 10 \times 10 = 100 \)  
(It’s a 1 with two zeros.)

This pattern continues all the way through.

**Ex. D:**

\[
\begin{array}{c}
3000 \\
\times 400 \\
\hline
1,200,000
\end{array}
\]

3 final zeros in 3000  
+ 2 final zeros in x 400  
5 final zeros in 1,200,000

**When multiplying with final (ending) zeros:**

First ignore the final zeros & multiply the rest of the digits to get a simpler product (3 x 4 = 12). Then you MUST write all the final zeros from all the factors at the end of that simpler product.  
(HINT: Count all the zeros in all the factors in the original problem.)

\[
\begin{array}{c}
3 \times 4 = 12 \text{ with ALL five zeros} = 1,200,000
\end{array}
\]

Answer: The product of 3000 and 400 is \( 1,200,000 \).

**Multiply:**

1) \( 400 \times 500 \)  
2) \( 6000 \times 70 \)  
3) \( 90 \times 80,000 \)
ESTIMATING PRODUCTS

First round **each** number to its **highest** place value (separately). This will get you:

1 digit (followed by zeros) \* 1 digit (followed by zeros)
That is, very easy multiplication facts with a bunch of zeros!

**Ex. A:** Estimate the product of 4287 \* 395  Think: 4000 \* 400 = 1,600,000

<table>
<thead>
<tr>
<th>NOTE: DO NOT round all numbers to the same place with products unless all the numbers have the same highest place value! Here if you round both numbers to the hundreds place (the highest common place value) you get:</th>
</tr>
</thead>
<tbody>
<tr>
<td>4300 * 400</td>
</tr>
</tbody>
</table>

Is this multiplication quick and easy enough to do in your head??  NO!!

The rules for estimating products and sums are different for a reason; to make the problems quick and easy to solve!

**Ex. B:** **Estimate** the product of 34 and 53. (See page 102, Ex. F)

Think 30 \* 50 = 1,500  

(3 \* 5 = 15 with 2 zeros)

Since 1,500 (the estimated product) is reasonably close to 1,802 (the exact answer) there doesn’t appear to be any major errors in the work.

| NOTE: Multiplication Estimates and Exact Products should have the same (or really close) highest place value and they should be in the same basic “neighborhood” of values. |

Estimate the following products:

1) 71 \* 28  
2) 127 \* 45  
3) 243 \* 352
What about multiplication with zeros in the middle??

Just follow the regular multiplication rules, but this can be made a little easier.

Understanding place value and why we use place holders in multiplication enables us to save some time and shorten the problem.

**Ex. A(i):** Many people would find the product of 203 and 406 as follows:

\[
\begin{array}{l}
\downarrow \\
203 \\
\times 406 \\
\hline
1218 \text{ (from } 6 \times \text{ every digit in } 203) \\
0000 \text{ (from } 0 \times \text{ every digit in } 203) \text{ – but this is just zero!} \\
+ 81200 \text{ (from } 4 \times \text{ every digit in } 203) \\
82418 \text{ is the product of } 203 \text{ and } 406.
\end{array}
\]

**Ex. A(ii):** Find the product of 203 and 406.

First distribute the 6 (from the 406) to every digit in the 203 (from the right)

\[
\begin{array}{l}
\downarrow \\
203 \\
\times 406 \\
\hline
8 \\
1218 \\
\hline
0
\end{array}
\]

Now multiply with the 4:

Distribute the 4 (from the 406) to every digit in the 203 (from the right) but record your answer next to your place holder zeros! Since the 4 (from the 406) represents hundreds, these answers will be correctly placed, starting in the hundreds place.

\[
\begin{array}{l}
\downarrow \\
203 \\
\times 406 \\
\hline
1218 \\
\hline
0
\end{array}
\]

When you get to multiplying by the zero (from the 406), 0 \times 203 = 0, so you can skip it. Because zero tens times anything = zero tens, **be sure to put a 0 place holder in the tens column**!

\[
\begin{array}{l}
\downarrow \\
203 \\
\times 406 \\
\hline
1218 \\
\hline
0
\end{array}
\]

82418 is the product of 203 and 406.
Find the products. (Check to see if the products seem reasonable by estimating.)

1) \[ 7 \quad 0 \quad 8 \]
   \[ \times \quad 9 \quad 0 \quad 4 \]

Estimate:

2) \[ 3 \quad 0 \quad 5 \]
   \[ \times \quad 6 \quad 0 \quad 4 \]

Estimate:

3) \[ 5 \quad 0 \quad 0 \quad 8 \]
   \[ \times \quad 9 \quad 0 \quad 0 \quad 2 \]

Estimate:

4) \[ 8 \quad 0 \quad 0 \quad 2 \]
   \[ \times \quad 4 \quad 0 \quad 0 \quad 1 \]

Estimate:
Homework 2.2

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Distributive Law</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Rounding Numbers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Estimate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Product</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Compare and contrast the Distributive Law and the Associative Law of Multiplication:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For questions 7 – 12, find the following products:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7)</td>
<td>(86 \times 59)</td>
<td>(401 \times 203)</td>
</tr>
<tr>
<td>8)</td>
<td>(100 \times 8000)</td>
<td>(4000 \times 50)</td>
</tr>
</tbody>
</table>

For questions 13 – 15, estimate the following products:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13)</td>
<td>(8124 \times 735)</td>
<td>(256 \times 972)</td>
</tr>
</tbody>
</table>

16) It takes 1560 plastic water bottles laid end to end to make a single layer of bottles around the perimeter of the Quad at NCC. When classes are in session, students, faculty and staff use enough bottles each day to build a wall of bottles 6 layers high around the Quad. There are roughly 280 days when classes are in session each year. How many bottles are being used each year?
Chapter 2.3

AREA AND PERIMETER

Units of measure: _____ This is one unit. It can represent 1 inch or 1 foot or 1 mile.

This is one square unit. It is a square that is 1 unit on each side.

**Perimeter** is the total number of units around the outside edge (or rim, as in perimeter) of the entire shape. That is why the formula for **perimeter is the sum of all sides**.

**Area** is the total number of squares that make up (or cover) the entire shape.

**Ex. A:**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>...</td>
<td>6...</td>
<td></td>
</tr>
</tbody>
</table>

Perimeter = 16 units
Area = 15 square units

**Ex. B:**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>...</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Perimeter = 16 units
Area = 14 square units

**Ex. C:**

Perimeter = 18 units
Area = 12 square units
AREA OF A RECTANGLE

AREA is a measurement of the size of a flat surface.

AREA asks the question “How many squares does it take to cover a particular shape?”

For this course we will only look at the areas of RECTANGLES and areas of shapes made by CONNECTING rectangles together or removing a rectangular shape from another rectangle.

Definition: A rectangle has three specific properties:
1. It is a four sided figure, AND
2. The two pairs of opposite sides of the rectangle have the same length, AND
3. The corners are perpendicular (form 90 degree angles).

\[
\text{AREA of a RECTANGLE} = \text{LENGTH} \times \text{WIDTH} \\
A = L \times W
\]

Why?

Because the length represents the total number of squares across one row and the width represents the total number of rows in a rectangle.

The formula for the area of a rectangle is not just \(A = L \times W\), it really means:

\[
\text{Area of a RECTANGLE} = \text{number of squares in each row} \times \text{the number of rows}
\]

NOTE: The label for any area has two parts: the first is the fact that we are using squares as the unit of measure; the second is the size of the square.

**Ex. A:**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 rows</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Area of this rectangle is:

\[
A = L \times W \\
A = 4 \text{ sq/row} \times 3 \text{ rows} \\
A = 12 \text{ sq. units* (or 12 units}^2) \\
\]

When you don’t know the unit name just use the word unit

*(where each little box is 1 sq. unit)*
Ex. B(i): Find the area of the following.
This question asks “how many squares are needed to cover this shape?”

\[
\begin{array}{c|c}
18 \text{ ft.} & 32 \text{ ft.} = 32 \text{ squares per row} \\
\hline
18 \text{ rows} & \text{The Area of this rectangle is:} \\
& A = L \times W \\
& A = 32 \text{ sq/row} \times 18 \text{ rows} \\
& A = 576 \text{ sq. ft.}
\end{array}
\]

Ex. B(ii) (cont): Another way to look at this problem considers splitting the shape into 2 pieces:

\[
\begin{array}{c|c}
18 \text{ ft} & 32 \text{ ft} \\
\hline
\text{Start with the original shape (which is one rectangle) and cut it into two rectangles. The size has not changed. The area for the whole rectangle has to be the same for the split rectangle.}
\end{array}
\]

\[
\begin{array}{c|c}
22 \text{ ft} & 10 \text{ ft} \\
\hline
\text{NOTE: 22 ft + 10 ft = 32 ft across the top}
\end{array}
\]

Looking at the figure this way there are 2 separate rectangles:

\[
\begin{align*}
\text{Area of rectangle A} &= 22 \text{ ft} \times 18 \text{ ft} = 396 \text{ sq ft} \\
\text{Area of rectangle B} &= 10 \text{ ft} \times 18 \text{ ft} = 180 \text{ sq ft} \\
\text{Area of the original rectangle} &= 576 \text{ sq ft}
\end{align*}
\]

Answer in a sentence: The area is 576 sq ft.

Being able to see the large rectangle broken up like this makes it easier to work on the problems that appear on the following page:
**Addition Method**

**Ex. A**: Find the area of the shape below. Another way to say this is, “How many squares big is the shape below? Or, “How many squares are needed to completely cover the shape below?”

BUT, we can split this picture up into rectangular pieces to find the area inside the given figure.

This can be done in two different ways using the Addition Method:

First Cut:

To find the area:
- a) Break the shape into two rectangles, A and B.
- b) Find the areas of A and B
- c) **Add** the areas of A and B to get the area of the original shape.

Notice we did not use the 32 ft (too long) or the 8 ft (too short). They weren’t necessary. You don’t always use every number in a problem.

Area of region A = 30 ft \( \times \) 18 ft = 540 sq ft
Area of region B = 10 ft \( \times \) 2 ft = +20 sq ft

560 sq ft
Ex. B: Find the area of the shape below. Another way to say this is, “How many squares big is the shape below? Or, “How many squares are needed to completely cover the shape below?”

(Addition Method continued) People “see” rectangles differently. In the previous solution we made two rectangles with a vertical cut. Below we make two rectangles with a horizontal cut.

To find the area:

a) Break the shape into two rectangles, A and B.
b) Find the areas of A and B
c) **Add** the areas of A and B to get the area of the original shape.

Area of A = 32 ft x 10 ft = 320 sq ft
Area of B = 30 ft x 8 ft = 240 sq ft

560 sq ft

Answer: The area is 560 sq ft.

NOTE: No matter how you break up the original shape to “see” the rectangles that make up that shape, the size of the shape is the same, so the area of the original shape must be the same.
**Subtraction Method**

**Ex. C:** Find the area of the shape below. Another way to say this is, “How many squares big is the shape below? Or, “How many squares are needed to completely cover the shape below?”

There is another way to look at this problem using subtraction rather than addition:

First consider the given figure:

Wouldn’t it have been nice if the figure (shape) were a nice rectangle?
Adding a (small) rectangle to the bottom corner of the figure (shape) will give us a perfect (bigger) rectangle to work with.

To find the area of the original figure (shape):

1. **Big** rectangle: 32 ft x 18 ft = 576 sq ft
2. **Small** rectangle: 8 ft x 2 ft = 16 sq ft
3. Area of the original figure: 560 sq ft

*Remember, the original figure (shape) you were given to work with was not a rectangle.*

**NOTE:** The answer is the same when calculated with the subtraction method as when we calculated the area inside the original figure by both of the addition method solutions.
Identify and label the rectangles in the following figures.

How might you use them to calculate areas?

1) 

![Diagram 1](image)

2) 

![Diagram 2](image)
3) 

4)
5) 

6)
In the following shapes, solve for the missing side:

7a) \[ \begin{array}{c}
4 \text{ ft.} \\
5 \text{ ft.} \\
x \text{ ft.} \\
11 \text{ ft.} \\
10 \text{ ft.} \\
\end{array} \]

7b) \[ \begin{array}{c}
14 \text{ ft.} \\
14 \text{ ft.} \\
17 \text{ ft.} \\
y \text{ ft.} \\
30 \text{ ft.} \\
31 \text{ ft.} \\
\end{array} \]

7c) \[ \begin{array}{c}
10 \text{ ft.} \\
11 \text{ ft.} \\
11 \text{ ft.} \\
z \text{ ft.} \\
20 \text{ ft.} \\
\end{array} \]

7d) \[ \begin{array}{c}
7 \text{ ft.} \\
9 \text{ ft.} \\
9 \text{ ft.} \\
q \text{ ft.} \\
16 \text{ ft.} \\
\end{array} \]
8) Find the area inside the given region three different ways.
Note: There are more than three ways but it starts to get complicated.
Try to use the fewest number of rectangles possible when calculating areas.

First think about the rectangles you can find in this picture (like in the previous problems).

See the Addition Method on pages 112-113 and the Subtraction Method on page 114 for help.

8a)

A look ahead or When will I ever need this?
Rooms are rectangular (or are two or more connected rectangles). When you order any type of flooring for a room (in other words, carpet or tiles that “cover the floor”) it is good to order the correct amount. Order too little and that’s a problem. Order too much and you will cover the floor but you will pay for extra carpeting, tiles, or wood flooring that you DO NOT NEED!
8b) 

8c)
Homework 2.3  Name: _______________________________________________________

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Area of a rectangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Solve for ( x ):</td>
<td><img src="image" alt="Problem Diagram" /></td>
<td></td>
</tr>
<tr>
<td>3) What steps would you take</td>
<td><img src="image" alt="Shape 1" /></td>
<td></td>
</tr>
<tr>
<td>4) What steps would you take</td>
<td><img src="image" alt="Shape 2" /></td>
<td></td>
</tr>
<tr>
<td>5) What steps would you take</td>
<td><img src="image" alt="Shape 3" /></td>
<td></td>
</tr>
</tbody>
</table>

For question 2, remember to show all work:

2) Solve for \( x \):
For questions 6-9 find the areas of the following shapes:

6) Solve with the **Addition** Method:

![Addition Method Diagram](image1)

7) Solve with the **Subtraction** Method:

![Subtraction Method Diagram](image2)

8)

![Diagram for Question 8](image3)

9)

![Diagram for Question 9](image4)
Chapter 2.4

DIVISION CONCEPTS

Definition: **Quotient**: The result of a division problem.

There are three Notations for Division:

1) We can divide from Left to Right.

\[ \frac{12}{4} = 3 \]

This is *calculator* division.

12 is the **dividend**, 4 is the **divisor**, and 3 is the **quotient**.

The check for this division is \(3 \times 4 = 12\). The 12 is the product of the two factors, 3 and 4.

When checking division, always multiply: \[\text{quotient} \times \text{divisor} = \text{product}\]

**NOTE**: The dividend is a product of the two factors (the quotient and the divisor).

2) We can divide Top to Bottom.

\[ \frac{12}{4} = 3 \]

This is *fraction* division.

12 is the **dividend**, 4 is the **divisor**, and 3 is the **quotient**.

Again, the check for this division is \(3 \times 4 = 12\): \[\text{quotient} \times \text{divisor} = \text{product}\]

3) We can divide Inside by Outside

\[ \underline{4} \overline{3} \]

This is called *long division*.

Long division is done with **paper & pencil**.

12 is the **dividend**, 4 is the **divisor**, and 3 is the **quotient**.

Again, the check for this division is \(3 \times 4 = 12\): \[\text{quotient} \times \text{divisor} = \text{product}\]

**NOTE**: \(+, -, \text{ and } \times\) have only two notations while \(\div\) is the only operation with 3 notations.

**NOTE**: Divisor is another way of saying factor in the division world.
Divison is repeated subtraction.

$12 \div 4 = 3$ can be thought of as…

12 items separated 4 items at a time. This is repeated subtraction…

$12 - 4 - 4 - 4$ (until the 12 is all used up).

This would result in 3 equal size groups:

or

**Division separates (breaks) numbers (things) into equal size groups.**

$12 \div 4 = 3$ can be thought of as…

12 items separated into 4 equal groups of 3 items in each

This is like dealing out 12 cards to 4 players, one at a time. When the 12 cards are all used up the result is each player receiving 3 cards.

Remember: check division by writing related multiplication: quotient $\times$ divisor $=$ product

$$12 \div 4 = 3 \ ?$$

Check: $3 \times 4 = 12$

$\text{Product} \div \text{factor} = \text{factor}$

NOTE: The factors are interchangeable in the multiplication sentence by the Commutative Law. This results in the same multiplication problem. The factors switch, but the product is the same.

What happens when you switch the factors in this division problem??

This gives us a different problem. It is a true statement that is related to the above 2 problems.

$$12 \div 4 = 3 \quad \text{Product} \div \text{factor} = \text{factor} \quad \text{(original problem)}$$

$$12 \div 3 = 4 \ ? \quad \text{Product} \div \text{factor} = \text{factor} \quad \text{(switched factors)}$$

Check both divisions with $3 \times 4 = 12$

These are the 3 related multiplication & division problems.
Sharing & Cutting Problems

Division helps us answer the following questions:
“How much in each…?” or “How much is each worth?” or “How big is each piece?”

NOTE: The portions or pieces MUST be equal in size!!!

When cutting or sharing something you must start with whatever is being cut or shared.

1) Ask “What is being cut or shared?” first and start your division with the number that represents the total amount you have to start.

When cutting or sharing something you must be given information about the pieces or portions.
Either you will be told how many pieces you need or how big each piece should be.

2) Ask what information was given about the pieces or portions. The divisor is the number that represents the given information you have about the pieces or portions.

3) The quotient is the other information about the pieces or portions.

Ex.A: A giant chocolate bar weighs 64 ounces. If you want to cut it up into small, equal sized pieces, so each piece weighs 2 ounces, how many pieces can you get from this giant bar?

64 (ounces) ÷ 2 (ounces) = 32

Total amount that is being cut or shared

Given information about the pieces:
Size per piece

The other information about the pieces:
Number of pieces

32 is not a complete answer. What does it mean? Since we were asked “how many pieces can you get?” so the quotient (the “other information about the pieces”) must be “number of pieces.”

Answer: You can get 32 pieces (each weighing 2 ounces) from a 64 ounce chocolate bar.
Ex.B: You and three friends want to share $25 you all won on a scratch-off game. If you all decide to share the money equally, how much money (whole dollars) will you and your friends each get (no coins)? Is there any money left over? If so, how much?

\[
\frac{25}{4 \text{ people}} = 6 \text{ r } 1
\]

$25 \div 4 \text{ people} = 6 \text{ r } 1

6 r 1 is not a complete answer. What does it mean? Since we were asked “How much does each friend get?” the quotient (the “other information about the portions”) tells us that each friend gets $6 and the remainder tells us there is $1 leftover. If 4 friends each get $6, then only $24 is accounted for, so $1 left over makes sense.

Answer: Each one gets $6 and there is $1 left over.

Answer the following questions:

1) An electrician has a roll of wire that is 120 feet long. If he needs equal length pieces that are each 5 feet long, how many pieces can he cut from this roll of wire?

2) An electrician has a roll of wire that is 120 feet long. If he needs 15 equal length pieces, how long should each piece be?
We can use related sentences to **solve** simple algebraic equations:

**Ex.A:** \(3 \cdot y = 15\) is really written as \(3y = 15\)

Solve this by dividing both sides of the equation by the number attached to \(y\) by multiplication:

\[
\begin{align*}
\frac{3y}{3} &= \frac{15}{3} \\
y &= 5
\end{align*}
\]

Divide both sides by 3 to get \(y\) alone on the left. The quotient on the right is \(15 \div 3 = 5\)

**Check** “missing number” problems with the following steps:

1) rewrite the original problem with ( ) in for \(y\) 
\(3(\ ) = 15\)
2) now substitute the value in for \(y\) in the ( )
\(3(5) = 15\)✓
3) DO THE MATH to see if it works!
So \(y = 5\) is a correct solution!

**Ex.B:** \(N \div 7 = 56\) can be written as \(\frac{N}{7} = 56\) or \(\frac{N}{7} = \frac{56}{1}\) so you can cross multiply:

\[N \times 1 = 56 \times 7\]
(this is the related multiplication)

\[N = 392\]

NOW: Check the answer: \((392) \div 7 = 56\) ✓ so the answer \(N = 392\) is correct!

Solve and check the following equations:

1) \(7x = 56\)

2) \(x \div 11 = 55\)
Do the Properties (Laws) of Multiplication work for Division?

Commutative Law: \[ 3 \times 4 = 4 \times 3 \]
\[ 12 = 12 \checkmark \]
Because: \[ 2 \neq 0.5 \]

\[ 4 \div 2 = 2 \div 4 \, ? \]
\[ \text{NO!} \]

Associative Law: \[ (3 \times 4) \times 5 = 3 \times (4 \times 5) \]
\[ 12 \times 5 = 3 \times 20 \]
\[ 60 = 60 \checkmark \]
Because: \[ 2 \neq 2 \div 4 \]

\[ (8 \div 4) \div 2 \neq 8 \div (4 \div 2) \, ? \]
\[ \text{NO!} \]

It’s obvious the properties (laws) of multiplication do NOT work for division.

Basic Division Rules

1. Any natural number ÷ itself = 1
\[ \frac{N}{N} = 1 \]
N ÷ N = 1
because \[ 1 \cdot N = N \]

2. Any whole number ÷ 1 = itself
\[ \frac{N}{1} = N \]
N ÷ 1 = N
because \[ N \cdot 1 = N \]

3. Zero ÷ any natural number = zero
\[ \frac{0}{N} = 0 \]
0 ÷ N = 0
because \[ 0 \cdot N = 0 \]

4. Any number ÷ zero is undefined
\[ \frac{N}{0} \neq 0 \]
If \[ N \div 0 = 0 \], then
\[ 0 \cdot 0 \neq N \]
This is impossible!
\[ \frac{N}{0} \neq N \]
If \[ N \div 0 = N \], then
\[ N \cdot 0 \neq N \]
This is impossible!

The problem is Zero times Anything = 0 so Zero times Anything will never equal N!

In either case, it is impossible to solve for N, so the problem is undefined!

Memory Trick: “Zero under the line is undefined”

Remember \[ N \div 0 \] spells NO! …meaning there is NO answer to this problem!

\[ N \div 0 \text{ is undefined!} \]
Homework 2.4  

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<tbody>
<tr>
<td>1) Quotient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Divisor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Factors</td>
<td></td>
<td></td>
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<tr>
<td>4) Product</td>
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<tr>
<td>Term</td>
<td>Official Definition</td>
<td>In your own words and/or example</td>
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<tr>
<td>5)</td>
<td>Notations for Division</td>
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<td>6)</td>
<td>Check for Division</td>
<td></td>
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<tr>
<td>7)</td>
<td>Related Multiplication &amp; Division</td>
<td></td>
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</tbody>
</table>
For questions 8 – 11, solve for \( x \) and check your work:

<p>| | | | |</p>
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<thead>
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</thead>
<tbody>
<tr>
<td>8)</td>
<td>( 6x = 54 )</td>
<td>9)</td>
<td>( 8x = 56 )</td>
</tr>
<tr>
<td>check:</td>
<td></td>
<td>check:</td>
<td></td>
</tr>
</tbody>
</table>

12) Is \( 15 \div 0 = 0 \)? Explain:

13) \( \frac{0}{51} = \) a) \( \frac{51}{0} = \) b) \( \frac{51}{51} = \) c) \( \frac{51}{1} = \) d) \( \frac{1}{1} = \)

14) Is \( 15 \div 0 = \frac{0}{15} \)? Explain:

15) Does \( \frac{1}{15} = 15 \)? Explain:
Chapter 2.5

LONG DIVISION

NOTE: Long division is done with paper & pencil.

Ex.A: Long division with a single digit divisor can be done with the times table in Appendix A.

There are 5 parts to long division:

1) Divide  2) Multiply  3) Subtract  4) Check  5) Bring Down
(One digit at a time!)

Memory trick: Does McDonalds Serve Cheese Burgers?

\[
\begin{array}{c}
5 \\
6 \overline{)3078} \\
\hline
\end{array}
\]

1. Divide: Start on the left of the number 3078 and use as many digits of 3078 to be able to divide by 6. In this case 3 is not enough but 30 is. \(30 \div 6 = 5\). Write the 5 above the 0 in 3078.

\[
\begin{array}{c}
5 \\
6 \overline{)3078} \\
-30 \\
\hline
30 \\
\end{array}
\]

2. Multiply: Multiply 5 \(\times\) 6 = 30, write the 30 underneath the 30.

\[
\begin{array}{c}
5 \\
6 \overline{)3078} \\
-30 \\
\hline
0 \\
\end{array}
\]

3. Subtract: Subtract 30 – 30 = 0 and write the 0 underneath.

\[
\begin{array}{c}
5 \\
6 \overline{)3078} \\
-30 \\
\hline
0 \\
\end{array}
\]

4. Check: Is 0 < 6? If it is, go to the next step. If it isn’t, there is an error somewhere. Check your subtraction, multiplication or division.

\[
\begin{array}{c}
5 \\
6 \overline{)3078} \\
-30 \\
\hline
07 \\
\end{array}
\]

5. Bring Down: Bring Down the 7 and start the process again this time using the 7.

\[
\begin{array}{c}
51 \\
6 \overline{)3078} \\
-30 \\
\hline
07 \\
\end{array}
\]

1. Divide: \(307 \div 6 = 1.16666\ldots\) The whole number 1 is all you need. Ignore all the decimals. Write the 1 above 7 (the number you brought down).

\[
\begin{array}{c}
51 \\
6 \overline{)3078} \\
-30 \\
\hline
07 \\
\end{array}
\]

2. Multiply: Multiply 1 \(\times\) 6 = 6, write the 6 underneath the 7.
3. **Subtract**: Subtract \( 7 - 6 = 1 \) and write the 1 underneath.

4. **Check**: Is \( 1 < 6 \)? If it is, go to the next step. If it isn’t, there is an error somewhere. Check your subtraction, multiplication or division.

5. **Bring Down**: Bring Down the 8 next to the 1 and start the process again this time using the 18.

1. **Divide**: \( 18 \div 6 = 3 \). Write the 3 above 8 (the number you brought down).

2. **Multiply**: Multiply \( 3 \times 6 = 18 \), write the 18 underneath the 18.

3. **Subtract**: Subtract \( 18 - 18 = 0 \) and write the 0 underneath.
4. **Check**: Is 0 < 6? If it is, go to the next step. If it isn’t, there is an error somewhere. Check your subtraction, multiplication or division.

5. **Bring Down**: There are no more digits in the number 3078 to bring down, so the work is done. The answer appears on top.

Check: 513 x 6 = 3078 ✓

Answer: The quotient of 3078 and 6 is 513.

**Check for long division**: Quotient x Divisor = Product or Dividend

(Answer x Outside number = Inside number*)

*NOTE: When checking long division, if the product does not match the inside number exactly, there must have been a remainder. Since the remainder came from a subtraction, ADD the remainder to get the exact inside number.

Find the following quotients.

1) \(3 \overline{) 1326}\)

2) \(4 \overline{) 8028}\)

3) \(7 \overline{) 7140}\)

4) \(9 \overline{) 1208}\)
Long division requires whole number remainders. Calculator division gives decimal answers.

NOTE: You can do long division with the help of a calculator, if you do it correctly:

\[
23 \quad ) \quad 3300 \\
\text{can be written as} \quad 3300 \div 23 \quad \text{which is a calculator problem:}
\]

NOTE: Long Division Style can be rewritten for the calculator if you write it correctly. Always remember:

**Inside number (3300) divided by the outside number (23)**

\[
3300 \div 23 = 143.4782608...
\]

143 is the whole number part and it is the **quotient**.

.4782608… means you have a remainder, but this is **not** the remainder. You have to find the remainder.

NOTE: Remainders are whole numbers!

Use the same process as in the long division in Ex. A on page 131, but let the calculator help.

1) **Divide**  2) **Multiply**  3) **Subtract**  4) **Check**

\[
\begin{align*}
\text{143} \\
23 \quad ) \quad 3300 \\
\end{align*}
\]

1. **Divide**: Using the calculator \(3300 \div 23 = 143.4782608\). The whole number 143 is the quotient and is written above the 3300. (NOTE: The 143 is written over the last 3 digits of the 3300.)

\[
\begin{align*}
\text{143} \\
23 \quad ) \quad 3300 \\
\underline{3289} \\
\end{align*}
\]

2. **Multiply**: Multiply 143 x 23 = 3289, write the 3289 underneath the 3300.

\[
\begin{align*}
\text{143} \\
23 \quad ) \quad 3300 \\
-3289 \\
\underline{11} \\
\end{align*}
\]

3. **Subtract**: Subtract 3300 – 3289 = 11. Write the 11 underneath. This is your remainder, so write r11 on top, next to the 143.

\[
\begin{align*}
\text{143 \hspace{1cm} r \hspace{0.5cm} 11} \\
23 \quad ) \quad 3300 \\
-3289 \\
\underline{11} \\
\end{align*}
\]

4. **Check**: Is 11 < 23? If it is then you are done. The calculator helps you do all the division in one big step. All the digits in the division (the first step) are used all at once. (NOTE: The remainder must be less than the divisor (outside number)!

\[
\text{Check: } 143 \times 23 + 11 = 3300 \checkmark \quad \text{(Quotient x Divisor + Remainder = Inside number)}
\]

Answer: 3300 divided by 23 is **143 r 11**.

What does all this mean? Since 143 is the quotient: If you took 3300 and subtracted 23 over and over and over again you could do this subtraction 143 times, with some leftovers (< 23).
Find the following quotients with the help of a calculator. Be sure to include remainders, if any are needed.

1) \[
46 \overline{) 2181}
\]

2) \[
63 \overline{) 7560}
\]

Check:

3) \[
72 \overline{) 7135}
\]

4) \[
129 \overline{) 12,308}
\]

Check:
Some Division Word Problems

1) If some number is divided by 18, the quotient is 71 and the remainder is 14. Find the number.

2) Consider two students with the following grades:

Which student do you think has a better chance of passing this course, the stronger test-taker who doesn’t like to do take-home assignments or go to lab? …or the one who can’t quite pass a test, but knows how to get extra help and does his/her work on time?

What grade does each student need on the final exam to pass the course (to get a 70 average)? What if the students want to qualify for the MAT001 exit exam? (A 75 average is required.)

HINT: See next page for information on Reverse Averages.

<table>
<thead>
<tr>
<th>Grades</th>
<th>Student X</th>
<th>Student Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>HW</td>
<td>92</td>
<td>65</td>
</tr>
<tr>
<td>Projects</td>
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<tr>
<td>Lab</td>
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<td>Test 1</td>
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<td>78</td>
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<tr>
<td>Test 2</td>
<td>68</td>
<td>82</td>
</tr>
<tr>
<td>Final Exam</td>
<td>??</td>
<td>??</td>
</tr>
</tbody>
</table>
Reverse Average Problems

If someone has an average of 80 it means that all of their grades are around an 80.

Sum the grades:

\[
\begin{align*}
80 + 75 + 85 + 70 + 90 &= 400 \text{ total points} \\
80 + 82 + 89 + 84 + 71 + 74 &= 480 \text{ total points} \\
80 + 80 + 79 + 80 + 81 + 80 + 76 + 84 &= 640 \text{ total points}
\end{align*}
\]

\[400 \div 5 = 80 \text{ average}\]
\[480 \div 6 = 80 \text{ average}\]
\[640 \div 8 = 80 \text{ average}\]

In each of the above examples if you divide the total number of points by the total number of grades the average comes out to be an 80.

Average grade = \[
\frac{\text{total number of points}}{\text{total number of grades}}
\]

NOTE: Cross multiply to get: Average x total number of grades = Total points

In the above examples all the test grades were known and we used those to find the average.

A Reverse Average Problem tells you the average and all the tests or grades except the last one. What is the minimum grade you will need to achieve that average?

**Ex. A:** These are your grades for the first 4 tests of the semester: 68, 77, 82, and 85. What grade do you need on the 5th test to get an 80 average for the semester?

\[
\begin{align*}
\text{Total points needed for average you want} - \text{Grades you already have} &= \text{Grade needed on the last test} \\
\text{Average desired x total number of tests (grades)} - \text{Sum the tests (grades) you have so far} &= \text{Grade needed on the last test} \\
80 \text{ avg x 5 tests} &= 400 - (68 + 77 + 82 + 85) = \text{The grade needed on the 5th test} \\
400 \text{ total points needed} - \text{312 points earned so far} &= 88 \text{ points needed on the 5th test}
\end{align*}
\]

88 is not a complete answer.

Answer the question, “What grade do you need on the 5th test to get an 80 average for the semester?”

**Answer:** I need an **88** on the 5th test to get an 80 average for the semester.
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<tbody>
<tr>
<td>1) Quotient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Divisor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Remainder</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Check for Division</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term:</td>
<td>Official Definition:</td>
<td>In your own words and/or example:</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>6) Product</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7) Reverse Average</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 8 – 11, find all the quotients and remainders (if any) and check your work:

8) \[
6 \overline{1218}
\]

9) \[
9 \overline{5436}
\]

10) \[
5 \overline{3297}
\]

11) \[
2 \overline{54,361}
\]
For questions 12 – 13, find all the quotients using a calculator with remainders (if any).

12) \[ \begin{array}{c}
37 \\
7989
\end{array} \]

Check

13) \[ \begin{array}{c}
31 \\
9672
\end{array} \]

Check

14) If a number is divided by 2 and the quotient is 6 and the remainder is 1, what is the number?

15) In a division problem where the divisor is 22, the quotient is 26 and the remainder is 21, find the missing number.
16) Suppose you have a job bagging marbles. You need to put 16 marbles in each bag to fill it. You start the day with 2010 marbles and pack the maximum number of bags possible. How many full bags did you pack? Were there any extra marbles? If so how many? What could you do with the extras if any?

17) One of the requirements to qualify for the MAT 001 exit exam (given at the end of the semester) is a 75 average for the course. If you have the grades listed in the table below what grade do you need on the final exam to get a final average of 75? Is it possible that this student will qualify for the MAT 001 exit exam? Why or why not?

<table>
<thead>
<tr>
<th>Grade</th>
<th>93</th>
</tr>
</thead>
<tbody>
<tr>
<td>HW</td>
<td></td>
</tr>
<tr>
<td>Project</td>
<td>80</td>
</tr>
<tr>
<td>LAB</td>
<td>75</td>
</tr>
<tr>
<td>Quiz</td>
<td>78</td>
</tr>
<tr>
<td>Test 1</td>
<td>72</td>
</tr>
<tr>
<td>Test 2</td>
<td>65</td>
</tr>
<tr>
<td>Final exam</td>
<td>??</td>
</tr>
</tbody>
</table>
Chapter 2.6
MULTIPLICATION AND DIVISION WORD PROBLEMS

Review the Problem Solving Techniques for Word Problems from Chapter 1 on pages 67 and 70.

Method A: Solving for the Unknown in an Equation

If you are given the value for one thing you can find the value for many of them with multiplication using the TOTAL VALUE FORMULA.

Start with the value for one thing, labeled as “Value per Item” (This is also called a Unit Size.)

Then multiply, matching the labels around the multiplication:

\[ \text{Value per Item} \times \text{Number of Items} = \text{Total Value} \]

(match these labels)

(notice: the value labels match too!)

The following is a situation to which everyone can relate:

**Ex.A:** If you make $9 per hour, how much do you make in 8 hours?

\[ \$ \text{ per hour} \times \text{Total hours worked} = \text{Total $$} \]

Notice the matching labels above!!!!

Use the words to place the numbers in the equation, then recopy the numbers and the math symbols (+, -, x, =) in the order they are written without the words and solve for the unknown in the equation.

\[ \$9 \text{ per hour} \times 8 \text{ hours worked} = \$ N \text{ total earned} \]

\[ 9 \times 8 = N \]
\[ 72 = N \]

72 is not the whole answer, it’s just part of the answer.

Answer the question, "How much do you make in 8 hours?"

**Answer:** You make $72 in 8 hours.
Ex. B: If the lab paid $12 for each calculator purchased online and $132 was spent, how many calculators were purchased?

Value per 1 Item $\times$ total number of Items $=$ Total Value

$\$12$ for 1 calculator $\times$ $N$ total calculators bought $= \$132$ total

\[
12 \times N = 132
\]

\[
12N = 132
\]

\[
\frac{12N}{12} = \frac{132}{12}
\]

\[
N = 11
\]

11 is not the whole answer, it’s just part of the answer.

Answer the question, “How many calculators were purchased?”

Answer: The lab purchased 11 calculators for $132.

1) This is a purchase order, a typical “real-life” example of the Total Value Formula

<table>
<thead>
<tr>
<th>Unit Price ($ per item)</th>
<th>Quantity</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>$31 per shirt</td>
<td>11 Shirts</td>
<td></td>
</tr>
<tr>
<td>$14 per tie</td>
<td>31 Ties</td>
<td></td>
</tr>
<tr>
<td>$29 per jacket</td>
<td>25 Jackets</td>
<td></td>
</tr>
<tr>
<td>$18 per belt</td>
<td>22 Belts</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL:</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) What is the total cost of the items purchased?

3) What is the **approximate** total cost?
Method B: Proportions

Proportions are two ratios that are equal. You solve proportions by cross multiplying.

Some people like to solve word problems that involve the “value for 1 thing” (or 1 item) with proportions.

Solving word problems using proportions takes the guess work out of solving one step multiplication and division problems.

There are 3 options to solve a simple multiplication or division word problem. That means you have a 1 out of 3 chance of guessing correctly. Those are terrible odds. Proportions (done correctly) will increase the odds of doing a problem correctly from $33\frac{1}{3}\%$ to 100%!

You have to pay close attention to the **LABELS** of the numbers. If you don’t, you may set up the proportion incorrectly, which will defeat the purpose.

Start with two numbers that “go together” to form the first ratio.

**Ex. A:** Write a ratio for a car that gets 20 miles per gallon.

20 miles per gallon has a hidden number in it. The word “gallon” is singular not plural and singular means the number 1.

20 miles per gallon is really 20 miles per 1 gallon.

$\frac{20 \text{ miles}}{1 \text{ gallon}}$ is the ratio for 20 miles per gallon. (Notice: the ratio is in fraction form.)

Once you can set up one ratio, you can set up the 2nd ratio making sure the labels on the top of both ratios are identical and the labels on the bottom of both ratios are identical.
Ex. B:

If you need to drive a total of 1500 miles for a business trip and you know your car gets 20 miles per gallon (mpg), how many gallons of gas will you need to purchase for this trip?

Make a list of the numbers in the word problem WITH THEIR LABELS! (Notice the 1 written before the word “gallon.”)

| 1500 miles | Make a list of the numbers in the word problem WITH THEIR LABELS! (Notice the 1 written before the word “gallon.”) |
| 20 miles per gallon | |
| N gallons | |

Write the first ratio and set up the second ratio, leaving space to write in the numbers.

The number 1500 is written next to the label “miles” and the variable N is next to the label “gallons.”

The problem is now an easy arithmetic problem to solve. Cross Multiply.

Solve for N by dividing both sides of the equation by 20 to get N alone.

Last step. 75 is not an answer. What was the question?

The N = 75 total gallons. Now go back and answer the original question, “How many gallons of gas will you need to purchase for this trip?”

Answer: You will need to purchase 75 gallons of gas for this trip.
BETTER BUY

You don’t want to spend more than is necessary for anything. To get the best value for your money, you need to be able to compare prices of the same items in a sensible way.

For example, when grocery shopping, you may find the same item in different size packages. You cannot compare the prices for these items IF they come in different size packages…

BUT you can compare prices if you get them on COMMON GROUND:

This common ground is the “unit price.”

<table>
<thead>
<tr>
<th>NOTE: Unit Price is the price for 1 ounce or 1 pound or 1 gallon, depending on the unit of measure for a particular item:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N per 1 pound ($ per lb)</td>
</tr>
</tbody>
</table>

Ex. A: Which box of cereal is the better buy, an 18oz box of cereal that costs $3.24 or a 22oz box of the SAME cereal that costs $4.18?

The “unit prices” can be found by dividing the cost of the box by the total number of units.

Here it is easier to think of the money in pennies instead of dollars to avoid the decimals:

Think: $3.24 = 324¢ and $4.18 = 418¢

Now, we want to find the price for 1 ounce of cereal in each box (price per ounce):

To do this, spread the total price out over the total ounces in the box (which is division):

Money divided by ounces gets you **Price per Ounce**:

\[
\frac{¢}{\text{oz}}
\]

\[
\frac{324¢}{18\text{oz}} = 18¢ \text{ per ounce}^* \\
\frac{418¢}{22\text{oz}} = 19¢ \text{ per ounce}
\]

*Usually the larger box is the “better buy,” but not always. As you see in this case, the smaller box (18oz) has the lower unit price, so it is the “better buy.”

Answer: The **smaller box (18oz)** is the better buy.
Solve the following problems. Some may require more than one step.

1) If you can type 74 words per minute, how long will it take you to type 17,316 words?

2) When the minimum wage is raised to $15 per hour in 2020 in New York State, how much will you make in one week if you work 40 hours for the week?

3) Put a fence around a yard that measures 12 yards by 8 yards. If fence costs $10 per yard, how much will it cost to fence in this yard (labor included)?

4) Install tile on a kitchen floor that measures 12 feet by 8 feet. If tile costs $10 per sq ft, how much will it cost to tile this floor (labor included)?
5) If a pen sells for 32¢, how many pens can you buy with $9 and how much change is left?

6) A car gets 31 miles per gallon. On a recent trip 42 gallons of gas were used. How many miles was the car driven on this trip?

7) The floor in the math lab is 28 feet long and 17 feet wide.
   a) Find the area of the floor.
   b) Find the perimeter of the lab.
   c) Floor tiles cost $18 a square foot. How much will it cost to tile the floor of the lab?
8) Which is a better buy, a box of 14 pens for $11.06 (1106¢) or a box of 26 pens (same pens) for $20.28 (2028¢)?

9) If five friends want to share 64 pieces of chocolate, how many whole pieces will each friend receive? Are there any leftover pieces? If so, how many?

10) A client is willing to spend up to $8000 to carpet the following room. Your boss wants you to go to the carpet store and get carpet samples to bring to this client. What price range can you consider for the carpeting? NOTE: Carpet prices are listed as $ per sq yd.
Homework 2.6  Name: 

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Sum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Product</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Quotient</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 5 - 6, remember to show all work:

5) Find the area and perimeter of a room that measures 18 ft wide and 30 ft long.

6) Crown molding costs $6 a foot ($6 per foot) and an additional $210 for labor. What is the total cost of putting crown molding in the room described in the previous question?
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>7) In your Reading 090 class, if you read 10 pages per day, 5 days a week for 13 weeks, will you accomplish the minimum goal of reading 1200 pages for the semester?</td>
<td></td>
</tr>
<tr>
<td>8) If you double the pages you read per day will that be enough? (Refer to question 7.)</td>
<td></td>
</tr>
<tr>
<td>9) Your job pays you $12 per hour. What is your gross paycheck (before taxes are taken out) if you work forty hours?</td>
<td>10) Your overtime rate of pay is $18 per hour for every hour you work beyond forty hours. How much will your overtime pay be if you worked 9 hours of overtime?</td>
</tr>
<tr>
<td>11) To answer this question, refer to questions 9 and 10: What is your gross pay if you work 49 hours in one week?</td>
<td>12) An auto mechanic earns $65,000 per year. What is her gross pay if she is paid weekly?</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>13) If you sail 168 miles round-trip from Huntington to Montauk and used 21 gallons of diesel fuel. How many miles per gallon will you get?</td>
<td></td>
</tr>
<tr>
<td>14) Diesel costs about $5 per gallon, what will 21 gallons of diesel cost?</td>
<td></td>
</tr>
<tr>
<td>15) Before leaving for the Poconos on vacation, the odometer read 86,938 miles. When I returned, the odometer read 87,372 miles. What was the average mpg if I used 14 gallons of gas?</td>
<td></td>
</tr>
<tr>
<td>16a) If you are paid once a week, how many paychecks do you receive in one year?</td>
<td>16b) If you earn $64,896 per year, what is your gross pay if you are paid once a week?</td>
</tr>
<tr>
<td>17a) If you are paid twice a month, how many paychecks do you receive in one year?</td>
<td>17b) If you earn $64,896 per year, what is your gross pay if you are paid twice a month?</td>
</tr>
<tr>
<td>18a) If you are paid every other week, how many paychecks do you receive in one year?</td>
<td>18b) If you earn $64,896 per year, what is your gross pay if you are paid every other week</td>
</tr>
</tbody>
</table>
Challenge Problems:
1) In an effort to budget for the school year (both fall and spring semesters) I need to find the cost for books and supplies. I registered for 4 courses for the fall semester and plan on taking 5 courses for the spring semester. I budgeted $90 per textbook for each course, $3 per notebook for each course, four packs of pens for the year at $6 each, and a ream of printing paper for each semester at $5 each. I also need a calculator but I can use it for both the fall and spring as long as I don’t lose it. If the calculator costs $15, how much money should I budget for textbooks and supplies for the school year?

2) A) Suppose you drive 2646 miles in a semester and your car gets 27 miles per gallon. Estimate your cost for gas for the semester if gas costs $2.89 per gallon?
Chapter 2.7

DIVISIBILITY TESTS

A number is divisible by another number if the other number goes in evenly without a remainder (or a decimal answer, if using a calculator).

12 is divisible by 4 because \(12 \div 4 = 3\) with no remainder (or decimal).
15 is not divisible by 7 because \(15 \div 7 = 2\) with remainder = 1 (or calculator answer = 7.5).

It is easy to check if a number is divisible by 2, 3, 5, 6, 9, and 10 by knowing a few rules. This will help when you are doing prime factorization and need to “break” down numbers quickly and easily (with no remainders or decimals).

**All numbers are divisible by 1:** Any Number \(\div 1 =\) that same number

A number is divisible by the following divisors:

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Test:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>If the ones digit is even (0, 2, 4, 6, 8).</td>
</tr>
<tr>
<td>3</td>
<td>If the sum of the digits is divisible by 3.</td>
</tr>
<tr>
<td>5</td>
<td>If the ones digit is 0 or 5.</td>
</tr>
<tr>
<td>6</td>
<td>If the ones digit is even and the sum of the digits is divisible by 3.</td>
</tr>
<tr>
<td>7</td>
<td>There is no test for 7. You just need to check on the calculator.</td>
</tr>
<tr>
<td>9</td>
<td>If the sum of the digits is divisible by 9.</td>
</tr>
<tr>
<td>10</td>
<td>If the ones digit is 0.</td>
</tr>
</tbody>
</table>

Write “yes” in the box if divisible, “no” if not.

<table>
<thead>
<tr>
<th></th>
<th>2 Ends in 0,2,4,6,8</th>
<th>Sum of digits</th>
<th>Sum/3</th>
<th>5 Ends in 0 or 5</th>
<th>(6) Divides by 2 and by 3</th>
<th>7 Just have to try!</th>
<th>Sum/9</th>
<th>10 Ends in 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex.A: 85</td>
<td>no</td>
<td>(8 + 5 = 13)</td>
<td>(13/3?)</td>
<td>No</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>(13/9?)</td>
</tr>
<tr>
<td>Ex.B: 13,251</td>
<td>no</td>
<td>(1+3+2+5+1 = 12)</td>
<td>(12/3?)</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>(12/9?)</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>555</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4444</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45,270</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>254,765</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Natural Numbers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Divisor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Factor</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4) Why is 1 the only natural number that is neither prime nor composite? 

5) What is the difference between a factor and a divisor?

6) Which of the following seven numbers are divisible by 2, 5, and 10 (at the same time)?

   245  700  350  84  63  574  105

7) Which of the following numbers are divisible by 7?

   245  112  700  350  84  63  105
Chapter 2.8

PRIME AND COMPOSITE NUMBERS

Definition: **Prime numbers**: Numbers with exactly 2 different factors, 1 and the number itself. Prime numbers **cannot** be broken down.

Definition: **Composite numbers**: Natural Numbers that have 3 or more factors. Composite numbers **can** be broken down, often in several ways.

NOTE: 1 is the only Natural number that is **neither** prime nor composite. You need two factors to be Prime and three or more factors to be Composite. Since the number 1 has only 1 factor it doesn’t qualify for either Prime or Composite.

The 1\textsuperscript{st} 10 prime numbers are:

\[2, 3, 5, 7, 11, 13, 17, 19, 23, 29 \ldots\]

NOTE: The Prime numbers go on forever.

The composite numbers are the numbers in between the prime numbers.

The 1\textsuperscript{st} 10 composite numbers are:

\[4, 6, 8, 9, 10, 12, 14, 15, 16, \text{ and } 18\]

PRIME FACTORIZATION

Definition: **Prime factorization**: Any composite number can be written as a product of its prime factors.

**Ex. A**: List all the factors of 15.

\[
\begin{align*}
15 \div 1 &= 15 \\
15 \div 2 &= 7.5 \\
15 \div 3 &= 5 \\
15 \div 5 &= 3 \text{, done!}
\end{align*}
\]

NOTE: To find factors, divide by natural numbers.

Answer: The factors of 15 are 1, 3, 5, and 15.

**Ex. B**: List all the prime factors of 15. Since the factors of 15 are 1, 3, 5, and 15:

Answer: The factors of 15 that are **prime** are just 3 and 5.
Ex. C: Write 12 as a product of its prime factors.

HINT: Use the list of the first 10 primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

12 = 3 x 4  3 is on the prime list but 4 is not.
Since 4 is not prime it can be written as a product of prime factors: 4 = 2 x 2, so replacing the 4:
12 = 3 x 2 x 2  12 is now written as a product of its prime factors.

What if you started differently? Suppose you wrote 12 as 2 x 6?

12 = 2 x 6  2 is on the prime list but 6 is not.
Since 6 is not prime it can be written as a product of prime factors: 6 = 2 x 3, so replacing the 6:
12 = 2 x 2 x 3  12 is now written as a product of its prime factors.

Answer: The prime factorization of 12 = 2 x 2 x 3

Prime Factorization - Method 1: Factor Trees

Ex. A(i): Write 280 as a product of its prime factors:  (Write a prime factorization for 280.)

Use the list of the first 10 prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Start with 2 and use your calculator to divide:

Factor Tree Method: 

280

\[ \begin{array}{c}
280 \\
\downarrow \quad \downarrow \\
\quad 140 \\
\quad \downarrow \quad \downarrow \\
\quad 2 \\
\quad \downarrow \\
\quad 70 \\
\quad \downarrow \\
\quad 2 \\
\quad \downarrow \\
\quad 35 \\
\quad \downarrow \\
\quad 5 \\
\quad \downarrow \\
\quad 7
\end{array} \]

280 ÷ 2 = 140
140 ÷ 2 = 70
70 ÷ 2 = 35
35 ÷ 2 = 17.5  2 is finished, try 3.
35 ÷ 3 = 11.666...  3 didn’t work try 5.
35 ÷ 5 = 7

Answer: The Prime Factorization for 280 = 2 x 2 x 2 x 5 x 7

NOTE: The factor tree method works well with the divisibility tests:

<table>
<thead>
<tr>
<th>2,</th>
<th>5,</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>even #s,</td>
<td>0 or 5</td>
<td>0</td>
</tr>
</tbody>
</table>

1st try the easy ones: Look at the 1’s digit and see:

Then try the sum test:  If the Sum ÷ 3, then the given # ÷ 3
(for 3 and 9 only)  If the Sum ÷ 9, then the given # ÷ 9

Last, try the annoying ones: Just divide out with a calculator or do long division.
(7, 11, and 13)
Prime Factorization - Method 2: Calculator

For those who don’t like the factor trees and divisibility tests, there is an alternative – for this method you need a list of the first 10 prime numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Using a calculator, just try dividing by the prime numbers, in order, to find prime factors.

Ex. A(ii): Write 280 as a product of its prime factors: (Write a prime factorization for 280.)

280 \div 2 = 140
140 \div 2 = 70
70 \div 2 = 35
35 \div 2 = 47.5
35 \div 3 = 11.666...
35 \div 5 = 7
7 \div 5 = 2.4
7 \div 7 = 1

Remember to try repeats! Notice here that the 2 is a factor again…
…and again!

When you get a quotient that is a decimal, cross out the prime you just tried and try dividing by the next prime number.

Be sure to use the same starting number in the division until you find a prime factor! (Here, start with 35 \div prime until 35 \div 7 works!)

When your quotient is 1 you are done! Now write your answer:

All the circled primes that you divided by (the divisors) are the prime factors that are multiplied in the prime factorization of the given number. The given number should be the product of all the primes you found. Be sure to check this product by multiplying!!

Answer: The Prime Factorization for 280 = 2 \times 2 \times 2 \times 5 \times 7

Find the Prime Factorizations of the following.

1) 42
2) 54
3) 273
4) 4004

See next page for an alternate method.
For the students who don’t like factor trees but like long division, there is a third method which also uses the list of prime numbers:

**Prime Factorization - Method 3: Layered Long Division**

**Layered Long Division:** For those who don’t like the factor trees and divisibility tests, there is an alternative – for this method you need a list of the prime numbers:

\[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \ldots\]

You will combine calculator work with a layering of long division problems to achieve the same results you would get using the factor trees.

**HOW TO DO layered long division:**

1. divide the given number by 2 on the calculator. Is the answer a whole number? If so, write the 2 as the divisor in the long division and the quotient from the calculator.

Now, try 2 again and keep repeating the previous step until you get a decimal answer, which means that the number you are trying to divide by 2 in the calculator is **not** divisible by 2.

Once the 2 doesn’t give you a whole number quotient, try dividing by 3, the next prime number. Continue through the prime numbers in this way until the quotient is itself a prime number. This is your signal to stop.

Remember to test for repeats!!

**NOTE:** In H.S. some of you may have learned a similar method called the “birthday cake” which has you dividing until the top of the layered division results in a 1. The 1 looks like a candle on top of a cake, thus the nickname! If you like to divide out until the last quotient is a 1, remember to NEVER use the 1 in your prime factorization! 1 is **NOT** a prime number!

**Ex A(iii):** Write 280 as a product of its prime factors: (Write a prime factorization for 280.)

\[
\begin{array}{c|c|c}
5 & 35 & 7 \\
2 & 70 & 35 \div 2 = 17.5; \quad 35 \div 3 = 11.666\ldots; \quad 35 \div 5 = 7 \checkmark \\
2 & 140 & 70 \div 2 = 35 \checkmark \\
2 & 280 & 140 \div 2 = 70 \checkmark \\
2 & 280 & 280 \div 2 = 140 \checkmark \\
\end{array}
\]

All of the prime numbers along the left side of the long division problems (divisors) and the final quotient* are the prime factors you need for your answer.

**Answer:** The Prime Factorization for 280 = \(2 \times 2 \times 2 \times 5 \times 7\)
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Product</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Factor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Prime Number</td>
<td></td>
<td></td>
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<tr>
<td>4) Composite Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Prime Factorization</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Can a number be both prime and composite? Explain:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7a) What is the only even prime number?

b) Why can no other even number be prime?

8) Indicate if the following numbers are prime, composite or neither:
   a) 31          b) 27          c) 2
   d) 91          e) 51          f) 1

For questions 8 – 11, find a prime factorization for the following numbers:

9) 186

10) 255

11) 1560

12) 6438
Chapter 2.9

EXPONENTS AND ROOTS

Exponents are a shorthand notation for repeated multiplication.

Ex.A: \( 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 \)

6 is called the exponent or the power.

2 is called the base.

Definition: Base: The base is the factor in the repeated multiplication.

Definition: Exponent: An exponent tells the number of times the base is written in the repeated multiplication.

2\(^6\) is read as “two to the 6\(^{th}\) (power)

5\(^4\) is read as “five to the 4\(^{th}\) (power)

5\(^3\) is read as “five to the 3\(^{rd}\) (power) or “5 cubed”

5\(^2\) is read as “five to the 2\(^{nd}\) (power) or “5 squared”

1) Write without exponents:
   a) \(4^3\)  
   b) \(5^4\)  
   c) \(2^7\)  
   d) \(6^3\)

2) Write with exponents:
   a) \(7 \times 7 \times 7 \times 7 \times 7\)  
   b) \(100 \times 100 \times 100 \times 100 \times 100\)

3) Write without exponents:
   a) \(3^5 \times 6^2\)  
   b) \(5 \times 2^4 \times 7^2\)

4) Evaluate the following expressions (1\(^{st}\) write without exponents):
   a) \(3^5 \times 6^2\)  
   b) \(5 \times 2^4 \times 7^2\)
Ex.B: Write the prime factorization for 164 written using exponents.
Use any one the three methods illustrated on pages 160 - 162 to find the prime factorization.

The prime factorization for 164 is $2 \times 2 \times 41$, so using exponents we get $164 = 2^2 \times 41$

Answer: The prime factorization for 164 written with exponents is $164 = 2^2 \times 41$

Find the prime factorization for the following numbers and express with exponents:

Note: 1st write without exponents and check the product. If it is correct, write with exponents.

Remember: Prime factorizations can be done with 3 different methods: see pages 160 – 162.

1) 18  
2) 25

3) 16  
4) 36
A look ahead or When will I ever need this?

Hate fractions? Mastering prime factorizations can help make fractions much easier to work with (without the need for a calculator). This (and understanding exponents) is also important when working with fractions in algebra. The rules you learn in this chapter as well as in the fraction chapter (along with prime factorizations) are the exact same rules that one uses to manipulate algebraic fractions, where calculators cannot be used.
PERFECT SQUARES

Definition: **Perfect square**: Any number multiplied by itself results in a **perfect square**.

Any base squared results in a **perfect square**.

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<tr>
<th>#</th>
<th>1</th>
<th>2</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>20</th>
<th>25</th>
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<tbody>
<tr>
<td>square</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>121</td>
<td>144</td>
<td>169</td>
<td>196</td>
<td>225</td>
<td>400</td>
<td>625</td>
</tr>
</tbody>
</table>

You can **reverse** this process by finding where a perfect square came from, that is, what number was multiplied by itself to get this perfect square.

This is called finding the **square root** of a perfect square.

\[ \sqrt{\text{perfect square}} \] is asking you to find the **base** of this inside expression where the perfect square inside the square root sign is thought of as \[ \text{perfect square} = \text{base}^2 \]

**Ex.A**: \[ \sqrt{64} \] can be thought of as \[ \sqrt{8^2} \] or \[ \sqrt{8 \cdot 8} \]. Since the base is 8, \[ \sqrt{64} = 8 \]

In a sense, the \( \sqrt{\cdot} \) and the exponent cancel each other out.

**NOTE**: \( \sqrt{\cdot} \) is called a square root sign or symbol or a radical sign.

The number inside the radical is called the **radicand**. In this course the radicand will always be a perfect square. This will not always be the case in any other course that uses radicals.

1) Find the following square roots:
   a) \( \sqrt{121} \)  
   b) \( \sqrt{100} \)  
   c) \( \sqrt{196} \)

2) a) Find the square of 4  
   b) Find the square root of 4

3) a) Find the square of 9  
   b) Find the square root of 9

4) a) Find the square of 100  
   b) Find the square root of 100
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Exponent</td>
<td></td>
<td></td>
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<tr>
<td>2) Repeated</td>
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</tr>
<tr>
<td>Multiplication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Powers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Base</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Squared</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Cubed</td>
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<td></td>
</tr>
</tbody>
</table>
7) Since $2 + 2$ and $2 \times 2$ are both equal to 4, can I make a rule for all natural numbers, that $n + n = n \times n$? Why or why not?

| For questions 8 – 11, write the following expressions without exponents: |
|---|---|
| 8) $4^3$ | 9) $67^3$ |
| 10) $2^2 \cdot 3^3 \cdot 4^4$ | 11) $3 \cdot 5 \cdot 11^2 \cdot 17^3$ |

| For questions 12 – 15, write the following expressions with exponents: |
|---|---|
| 12) $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ | 13) $81 \cdot 81 \cdot 81 \cdot 81$ |
| 14) $4 \cdot 4 \cdot 4 \cdot 8 \cdot 8$ | 15) $2 \cdot 2 \cdot 2 \cdot 10 \cdot 10$ |

| For questions 16 – 17, evaluate the following expressions: |
|---|---|
| 16) $8 \cdot 8 \cdot 8 \cdot 8$ | 17) $2^2 \cdot 3 \cdot 5^2$ |
For questions 18 – 19, write the prime factorization of the following numbers using exponents:

<table>
<thead>
<tr>
<th></th>
<th>1372</th>
<th>9000</th>
</tr>
</thead>
</table>

20) What is the square of 25?

21) What is the square root of 25?

22) Evaluate: $\sqrt{16}$

23) Evaluate: $16^2$

24) What is the square of 225?

25) What is the square root of 225?

26) Evaluate: $\sqrt{169}$

27) Evaluate: $169^2$
Chapter 2.10

ORDER OF OPERATIONS

The phrase "order of operations" should make you think of PEMDAS – a trick for remembering which operation must go 1st, then 2nd, etc.

What does PEMDAS really stand for?

<table>
<thead>
<tr>
<th>P</th>
<th>Work the problem inside the Parentheses 1st to get a single value. Note: if there is no operation to complete inside, it means multiply: Ex. (2)(3) or 2(3) means 2 x 3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Evaluate Exponents and roots next.</td>
</tr>
<tr>
<td>M/D</td>
<td>Means Multiplication and/or Division, whatever you see first moving from the left to the right through the expression.</td>
</tr>
<tr>
<td>A/S</td>
<td>Means Addition and/or Subtraction, whatever you see first moving from the left to the right through the expression. Remember: Add and Subtract LAST from left to right.</td>
</tr>
</tbody>
</table>

A helpful technique when simplifying or evaluating complicated expressions involving “order of operations” and “PEMDAS” is called the Window Shade Technique:

How to use the Window Shade Technique:

Ask PEMDAS what to do 1st. Mark it and then do it (and only do that operation, nothing else!!). Write the answer to that step directly below on the next line.

Here comes the window shade part…RECOPY the rest of the problem directly below on the next line exactly as you see it!

Now treat this next line as a new problem.

Ask PEMDAS what to do 1st. Mark it, etc…

Ex. A: These look similar but they have different outcomes.

| (3 + 8)^2 | (3 + 8^2) | (3^2 + 8^2) |
| (11)^2 | (8x8) | (3x3) | (8x8) |
| EXP: 121 | EXP: 3 + 64 | EXP: 9 + 64 |
| A/S: 67 | A/S: 73 |

Answer: (3 + 8)^2 = 121  Answer: (3 + 8^2) = 67  Answer: (3^2 + 8^2) = 73
Ex. B: These look similar but they have different outcomes.

**NOTE:** We expect to see a single value under a radical sign (as in the sample on the right). ( ) is understood to be around the numbers under the radical in the sample on the left.

\[
\sqrt{36 + 64} \quad \rightarrow \quad \sqrt{100} \\
\sqrt{36 + 64} \quad \rightarrow \quad 6 + 8 \\
\]

Answer: 10  
Answer: 14

Ex. C: These look similar but they have different outcomes.

\[
42 \div 6 \times 7 \\ M/D: \quad \rightarrow \quad 7 \times 7 \\
42 \div (6 \times 7) \\ M/D: \quad \rightarrow \quad 42 \div 42 \\
\]

Answer: 49  
Answer: 1

Ex. D: These look similar but they have different outcomes.

\[
11 - 3 + 8 \\
A/S: \quad \rightarrow \quad 8 + 8 \\
11 - (3 + 8) \\
A/S: \quad \rightarrow \quad 11 - 11 \\
\]

Answer: 16  
Answer: 0

Ex. E: These look similar but they have different outcomes.

\[
23 - 2(2 + 1)^2 \\
( ): \quad \rightarrow \quad 23 - 2(3)^2 \\
EXP: \quad \rightarrow \quad 23 - 2x9 \\
M/D: \quad \rightarrow \quad 23 - 18 \\
A/S: \quad \rightarrow \quad 5 \\
( ): \quad \rightarrow \quad (23 - 2)(2 + 1)^2 \\
EXP: \quad \rightarrow \quad (21)(3)^2 \\
M/D: \quad \rightarrow \quad (21)9 \\
A/S: \quad \rightarrow \quad 189 \\
\]
Problem Set I (basic):
1) $12 + 5(4)$  
2) $8 - 2 + 3$  
3) $18 \div 2 \cdot 3$

4) $5(6) - 12 \div 4 + 3$  
5) $\sqrt{256} - 2^4 + 1^{11}$  
6) $6^2 \div 2 - 1 + \sqrt{9}$

Problem Set II (parentheses):
7) $(3 \cdot 4)^2$  
8) $3 \cdot 4^2$  
9) $3^2 \cdot 4^2$

10) $(3 + 4)^2$  
11) $3 + 4^2$  
12) $3^2 + 4^2$
Problem Set III:

13) \( \sqrt{9} (10 ÷ 2) + 4^2 \)

14) \( 5^2 - 3(8 - 6) ÷ 2 \)

15) \( \sqrt{5^2 - 3^2} - 2 + 6^2 ÷ 9 \)

16) \( 4^3 ÷ (5 - 3) - \sqrt{9} + (8 ÷ 2^2) \)

Challenge: \( \sqrt{100} [ 6^2 - 4(5) ] - 2^3 + (2^4 ÷ 8) - 2\sqrt{25} \)

A look ahead or When will I ever need this?

Order of operations is a topic that applies to all math courses and all applications of mathematics. Complex formulas you will encounter in Statistics and Algebra courses require an understanding of order of operations.
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

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<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Order of Operations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Sum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Difference</td>
<td></td>
<td></td>
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<tr>
<td>4) Product</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Quotient</td>
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<tr>
<td>6) Exponent</td>
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<tr>
<td>Term:</td>
<td>Official Definition:</td>
<td>In your own words and/or example:</td>
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<tr>
<td>7) Square Root</td>
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</tbody>
</table>

For questions 8 – 15, perform the indicated operations (evaluate):

8) \[14 - 10 + 4\]  
9) \[24 ÷ 3 \times 8\]

10) \[20 + \sqrt{25} - 8 \times 3\]  
11) \[18 ÷ 9 \times 3 + 4 - 7\]

12) \[(12^2 - 4 \times 11) ÷ \sqrt{100}\]  
13) \[(3 + \sqrt{4})^2 + 6 \times 12 - 8\]
14) \[ 18 - (1 + 3^2) + 2 \times 7 \]

15) \[ 28 + 4 - (2 \times 7) - 3^2 \]

Challenge Problems:

For questions 16 – 17, perform the indicated operations (evaluate):

16) \[ 6 \times 7 \div 2 - \sqrt{12^2 + 5^2} + 8 \]

17) \[ \sqrt{(3 + 7)^2 - 6^2 - 2^3} + 10 \div (2 \times 5) \]
**Area**

**Definition:** Area measures the size of a flat surface using square units. The formula for the area of a rectangle:

\[
\text{Total # of squares} = \text{# of squares per row} \times \text{# of rows}
\]

**Example:**

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<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
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</tbody>
</table>

This rectangle is 12 square units.

**Distributive Law**

**Definition:** Adding two numbers first then multiplying by a factor is the same as multiplying each number by the factor and then adding those products.

**Example:**

\[
12 \times (2 + 4) = (12 \times 2) + (12 \times 4)
\]

Check: 12 x 6 = 24 + 48

\[72 = 72 \checkmark\]

**Divisor**

**Definition:** A divisor is the number that is being repeatedly subtracted from an original number to determine how many complete groups are contained in the original number.

**Example:** In the division sentence:

\[
12 \div 3 = 4
\]

3 is the divisor and there are 4 full groups of 3 contained in the number 12.

**Exponents (Powers)**

**Definition:** The number in an exponential expression that tells you how many times to repeatedly multiply the base by itself.

**Example:** In the exponential expression, \(5^4\), the exponent is 4, so the factor 5 is multiplied by itself 4 times.

\[5 \times 5 \times 5 \times 5 = 625\]

**Factor**

**Definition:** Factors are the numbers being multiplied.

**Example:** In the multiplication sentence:

\[12 \times 3 = 36\]

12 and 3 are two factors of 36.

**Composite Number**

**Definition:** A composite number is a number that has 3 or more factors.

**Example:** The factors of 9 are 1, 3, and 9, so 9 is a composite number because it has at least 3 factors.

**Perfect Square**

**Definition:** A perfect square is a result of multiplying any whole number by itself.

**Example:** \(0^2 = 0\), \(1^2 = 1\), \(2^2 = 4\), \(3^2 = 9\), \(4^2 = 16\)...

The perfect squares are 0, 1, 4, 9, 16...

**Identity Property of Multiplication**

**Definition:** Multiplying a number by 1 gives the same number as the product.

**Example:** 18 x 1 = 18, (18 kept its “identity”).

**Base**

**Definition:** The number being repeatedly multiplied in an exponential expression.

**Example:** In the exponential expression, \(5^4\), the base is 5, the repeated factor.

**Cube**

**Definition:** A cube of a number is the number times itself three times.

**Example:** The cube of 4 is \(4^3 = 4 \times 4 \times 4 = 64\).
Powers
Definition: (See exponents).

Prime Factorization
Definition: The process of writing a composite number as a product of prime factors.
Example: 12 can be written as the product of the prime factors 2 and 3.
12 = 2 x 2 x 3  (Think of “Factor Trees”).

Prime Factorization with Exponents
Definition: Using exponents for repeated prime factors instead of writing them separately.
Example: 12 = 2² x 3

Prime Number
Definition: A prime number has exactly two different factors, 1 and itself.
Example: The factors of 2 are 1 and 2 so 2 is a prime number.
The first ten prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

Product
Definition: A product is the answer to a multiplication problem.
Example: In the multiplication sentence 4 x 5 = 20, the product is 20.

Quotient
Definition: A quotient is the answer to a division problem.
Example: In the division sentence, 20 ÷ 5 = 4, the quotient is 4.

Remainder
Definition: A remainder is the leftover quantity in a division problem.
Example: In the following division sentence 23 ÷ 5 = 4 r 3, the remainder is 3.

Square
Definition: A square of a number is the number squared, that is, the number times itself.
Example: The Square of 36 = 36² = 1296.
Remember: 36² = 36 x 36

Square Root
Definition: The square root of a given number asks you to determine what number must be multiplied by itself to get the number under the radical sign.
Example: The Square Root of 36 = √36 = 6
Remember: √36 = √6 • 6 = 6

Important Ideas and Concepts You Need to Know

Better Buy: Any item identical in quantity and quality to another, but costs less, is a Better Buy. See pages 149 and 356.

Check for Long Division:
Quotient x Divisor + Remainder = Dividend
Top Number x Outside Number + Remainder = Inside Number
See page 135.

Cutting/Separating into Equal Size Pieces: When cutting or separating something into equal size pieces or portions, it is being “evenly divided.” Start with the thing that is being cut or separated followed by the division sign followed by the divisor which is either the number of equal portions or the size of each of portion.

(total amount) ÷ (amount in each portion) = (total number of portions)
Or
(total amount) ÷ (total number of portions) = (amount in each portion)
See page 124-126, 184, 186, 275, 276.

Division Notations: See Notations for Division.
Division Rules/Facts:

1) Any Number Divided by One is that Number
   \[ N ÷ 1 = N \]
   \[ \frac{N}{1} = N \]
   \[ 1 \div N = \frac{N}{1} \]

2) Zero divided by Anything is Zero
   \[ 0 ÷ N = 0 \]
   \[ \frac{0}{N} = 0 \]
   \[ N \div 0^- \]
   Check: \[ 0 \times N = 0 \]  
   \[ 0 \times N = 0 \]  
   \[ 0 \times N = 0 \]

3) Anything divided by Zero is **UNDEFINED**
   \[ N ÷ 0 = \text{Undefined} \]
   \[ \frac{N}{0} = \text{Undefined} \]
   \[ 0 \div N \]
   All division needs to be checked using Quotient x Divisor = Dividend (Product)
   
   Most people want to say \( \frac{N}{0} = 0 \), but the check shows: \( 0 \times 0 = N \)? Impossible!
   
   Then people want to say \( \frac{N}{0} = N \), but using the check gives: \( N \times 0 = N \)? Impossible!
   
   Then people want to say \( \frac{N}{0} = 1 \), but using the check gives: \( 1 \times 0 = N \)? Impossible!
   
   If \( \frac{N}{0} = \text{something} \), the check shows \( 0 \times \text{something} = N \), which is impossible!

   **That is why Division by Zero is UNDEFINED!**

*See page 128.*

**Estimate:** To find an approximate answer to any mathematical problem by first rounding **all** numbers (using the **highest** place value) then performing the indicated operation with the rounded numbers.

*See page 104.*

**Multiplication by 0 (Zero Property) and Multiplication by 1 (Identity Property):**

- Anything times Zero is Zero and Zero times Anything is Zero:
  \[ N \times 0 = 0 \]
  \[ 0 \times N = 0 \]

- Any Number times One is that Number and One times Any Number is that Number:
  \[ 1 \times N = N \]
  \[ N \times 1 = N \]

*See pages 97.*

**Notations for Division:** There are three different notations for division. In other words, there are three different ways to write a division sentence. **For Example:** fifty-six divided by eight equals seven.

1) \( 56 ÷ 8 = 7 \)  
2) \( \frac{56}{8} = 7 \)  
3) \( 56 \div 8 = 7 \)

Left ÷ Right  
56 ÷ 8 = Top ÷ Bottom  
56 ÷ 8 = Inside ÷ Outside

*See page 123.*
Order of Operations aka PEMDAS \[ P \ E \ (M/D) \ (A/S) \] – Multi-step problems with many different operations are to be done in a specific order.

**Step 1:** \( P \) stands for (Parentheses), [brackets], or \{braces\} - (grouping symbols) - Any mathematical operations inside grouping symbols are evaluated first.

*Example:* \( 8 \times (3 + 1) = 8 \times 4 = 32 \)

**Step 2:** \( E \) stands for Exponents - Any numbers with exponents (including square root symbols) are done second.

*Example:* \( 4 + 2^3 = 4 + 8 = 12 \) \( \quad 5 + \sqrt{36} = 5 + 6 = 11 \)

**Step 3:** \( M/D \) stands for Multiplication and Division - Any multiplication and division problems are done third, working in order from left to right. If you see multiplication first (on the left) do that, but if division is first (on the left) do that.

*Example:* \( 42 \div 7 \times 6 = 6 \times 6 = 36 \) \( \quad 3 \times 4 \div 2 = 12 \div 2 = 6 \)

**Step 4:** \( A/S \) stands for Addition and Subtraction – Adding and Subtracting are done are done last, working in order from left to right. If you see addition first (on the left) do that, but if subtraction is first (on the left) do that.

*Example:* \( 11 - 3 + 8 = 8 + 8 = 16 \) \( \quad 5 + 6 - 11 = 11 - 11 = 0 \)

*See page 175.*

**PEMDAS:** See Order of Operations above.

**Related Multiplication and Division Sentences:** Using a given multiplication sentence there are two related division sentences, both starting with the product.

*For Example:* From the multiplication sentence \( 8 \times 7 = 56 \), where 56 is the product, we can write:

\[ 56 \div 8 = 7 \quad \text{and} \quad 56 \div 7 = 8 \]

*See page 124.*

**Reverse Average** is the process of finding a specific test score required to achieve a desired average grade for a course.

*See page 139.*

**Sharing and Cutting Problems:** See Cutting/Separating into Equal Size Pieces.

**Total Value Problems:**

\( \text{(amount for one item)} \times \text{(total number of items)} = \text{(total value of all the items together)} \)

*See page 145-146*

**Unit Cost/Unit Price:** Unit Cost is found by the formula:

\( \text{Total Cost} \div \text{Total Number of Units} = \text{Cost for One Unit} \)

*See page 149.*

**Zero Property:** See Multiplication by 0 above.
For questions 1 – 2, identify the factors and product:

1) \[ 185 = 37 \times 5 \]

2) \[
\begin{align*}
8007 \\
\times 1003 \\
\hline
8,031,021
\end{align*}
\]

3) What happens when you multiply any number by 0?

4) What’s the largest factor of any number?

For questions 5 – 7, find all the factors of the following numbers:

5) 20:

6) 30:

7) 49:

For questions 8 – 9, Identify the following laws:

8) \[(a \times b) \times c = a \times (b \times c)\]

9) \[1 \times 2,345,671,895 = 2,345,671,895\]

10) Complete the following statement using the Commutative Law of Multiplication:

\[413 \times 398 = \]
For questions 1 – 6, find the following products:

<table>
<thead>
<tr>
<th></th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>(328 \times 52)</td>
</tr>
<tr>
<td>2)</td>
<td>(802 \times 307)</td>
</tr>
<tr>
<td>3)</td>
<td>(4003 \times 906)</td>
</tr>
<tr>
<td>4)</td>
<td>(600 \times 90)</td>
</tr>
<tr>
<td>5)</td>
<td>(200 \times 3000)</td>
</tr>
<tr>
<td>6)</td>
<td>(7000 \times 800)</td>
</tr>
</tbody>
</table>

7) Complete the following according to the distributive law:

\[12 \times (7 + 9) =\]

For questions 8 – 10, **estimate** the following products:

<table>
<thead>
<tr>
<th></th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>8)</td>
<td>(4128 \times 356)</td>
</tr>
<tr>
<td>9)</td>
<td>(625 \times 792)</td>
</tr>
<tr>
<td>10)</td>
<td>(1238 \times 958)</td>
</tr>
</tbody>
</table>

11) Today it costs $9 for a pack of cigarettes. Assuming this price never changes (really?!?), how much will it cost you to smoke a pack a day for the next 62 years? **NOTE:** Ignore leap years!
1) Solve for $x$:

\[
\begin{array}{c}
\text{7 ft} \\
9 \text{ ft} \\
\text{x ft} \\
9 \text{ ft}
\end{array}
\]

\[
\begin{array}{c}
\text{18 ft} \\
\text{20 ft}
\end{array}
\]

2) Find the area of the following shape:

\[
\begin{array}{c}
\text{7 ft} \\
9 \text{ ft} \\
13 \text{ ft} \\
11 \text{ ft}
\end{array}
\]

\[
\begin{array}{c}
\text{20 ft} \\
\text{20 ft}
\end{array}
\]

3) Find the area of the following shape:

\[
\begin{array}{c}
\text{20 in} \\
11 \text{ in} \\
\text{8 in} \\
\text{3 in}
\end{array}
\]

4) Find the area of the shaded region:

\[
\begin{array}{c}
\text{7 ft} \\
3 \text{ ft} \\
4 \text{ ft}
\end{array}
\]

\[
\begin{array}{c}
\text{10 ft} \\
\text{17 ft}
\end{array}
\]
1) Write a check for the following division problem:  \[ 15 \div 3 = 5 \]

2) Write 2 division problems related to the multiplication problem:  \[ 3 \times 5 = 15 \]

3) \[ 2 \times 3 = 6 \] is a check for the division problem \[ 6 \div 3 = 2 \] but it is also a check for another related division problem. What is that other related division?

For questions 4–7, solve for \( x \) and check your work:

<table>
<thead>
<tr>
<th>Question</th>
<th>Equation</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>4)</td>
<td>[ 3x = 39 ]</td>
<td>[ x = \frac{39}{3} = 13 ]</td>
</tr>
<tr>
<td>5)</td>
<td>[ 6x = 48 ]</td>
<td>[ x = \frac{48}{6} = 8 ]</td>
</tr>
<tr>
<td>6)</td>
<td>[ x \div 5 = 35 ]</td>
<td>[ x = 35 \times 5 = 175 ]</td>
</tr>
<tr>
<td>7)</td>
<td>[ x \div 20 = 400 ]</td>
<td>[ x = 400 \times 20 = 8000 ]</td>
</tr>
</tbody>
</table>

8) Why is \( 0 \div 15 = 0 \)? Use common sense.

9) Is division commutative? Why or why not.
For questions 1 – 5, find all the quotients and remainders (if any) and check your work:

<table>
<thead>
<tr>
<th>Question</th>
<th>Expression</th>
<th>Quotient</th>
<th>Remainder</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>$4 \div 3208$</td>
<td>$802$</td>
<td>$0$</td>
<td>✓</td>
</tr>
<tr>
<td>2)</td>
<td>$3 \div 1110$</td>
<td>$370$</td>
<td>$0$</td>
<td>✓</td>
</tr>
<tr>
<td>3)</td>
<td>$11 \div 2950$</td>
<td>$268$</td>
<td>$2$</td>
<td>✓</td>
</tr>
<tr>
<td>4)</td>
<td>$76 \div 8360$</td>
<td>$108$</td>
<td>$40$</td>
<td>✓</td>
</tr>
<tr>
<td>5)</td>
<td>$43 \div 1244$</td>
<td>$28$</td>
<td>$24$</td>
<td>✓</td>
</tr>
<tr>
<td>6)</td>
<td>$34 \div 2244$</td>
<td>$66$</td>
<td>$2$</td>
<td>✓</td>
</tr>
</tbody>
</table>
7) If a number is divided by 4 and the quotient is 5 and the remainder is 0, what is the number?

8) When a division problem is finished the quotient is 15, the remainder is 10, and the divisor is 12. With what number did this division problem begin?

9) An entire elementary school is going on a field trip. There are 462 students, faculty and parent volunteers going. Each bus seats 44 passengers besides the bus driver. How many buses will be needed? Ten extra students as well as 10 extra parents show up on the day of the trip. If you want to allow as many people to attend as possible, what would you tell them?

10) Your friend wants a 4.0 GPA in her PSY 203 class, which means she needs an A, or at least a 90% average, for the course. If her grades are as follows, what grade does she need on the final exam to get the 4.0 GPA? Do you have any advice for her?

<table>
<thead>
<tr>
<th>Grades:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Participation</td>
<td>70</td>
</tr>
<tr>
<td>Attendance</td>
<td>65</td>
</tr>
<tr>
<td>Projects</td>
<td>75</td>
</tr>
<tr>
<td>Test 1</td>
<td>95</td>
</tr>
<tr>
<td>Test 2</td>
<td>92</td>
</tr>
<tr>
<td>Final exam</td>
<td>??</td>
</tr>
</tbody>
</table>
1a) If you drive 203 miles round-trip between NCC and the Montauk Point Lighthouse and your car gets 21 miles per gallon (mpg), approximately how many gallons of gas will you need to drive the round trip?

1b) If gas costs about $4 per gallon, approximately how much did you pay for the gas for the round trip to Montauk?

2) Before leaving for a visit to Binghamton U, the odometer read 111,938 miles. After returning, the odometer read 112,358 miles. What was the average mpg if 20 gallons of gas was used?

3a) Find the area and perimeter of a room that measures 8 ft wide and 13 ft long.

3b) Ceramic tile costs $6 a square foot ($6 per sq ft) and an additional $300 for labor. What is the total cost of putting ceramic tile in the room (described in problem 3a)?

4) Suppose you had a friend registering for BEP 090 Reading class, next semester. How many pages would you advise your friend to read every day to accomplish the minimum goal of reading 1500 pages for the full 15 week semester, assuming you strongly suggested they start reading right away?
1) Which of the following eight numbers are divisible by both 2 and 5 (at the same time)?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>420</td>
<td>785</td>
<td>42,367,943,520</td>
</tr>
<tr>
<td>90</td>
<td>43</td>
<td>2,365,712</td>
</tr>
<tr>
<td>11</td>
<td>3800</td>
<td>123,456,789</td>
</tr>
</tbody>
</table>

2) If a number is divisible by both 2 and 5, is it automatically divisible by 10? Why? (Think about the factors of 10.)

3) Indicate whether or not the numbers in the top row are divisible by the numbers in the left column.

<table>
<thead>
<tr>
<th></th>
<th>4554</th>
<th>1950</th>
<th>33,121</th>
<th>12,133</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1) What is the only natural number that is neither prime nor composite? Explain:

2) What types of numbers can be written with prime factorization? Explain:

For questions 8 – 11, find a prime factorization for the following numbers:

3) 180

4) 455

5) 1564

6) 19,404
1) Rewrite the following expressions in a simpler way, then evaluate both of them, and then compare them:

\[2 + 2 + 2 + 2 + 2 + 2 + 2\]
\[2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2\]

For questions 2 – 5, write the following expressions **without** exponents:

<table>
<thead>
<tr>
<th>2) (5^5)</th>
<th>3) (4^7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4) (3^4 \cdot 8^3 \cdot 12)</td>
<td>5) (16^4 \cdot 31^2)</td>
</tr>
</tbody>
</table>

For questions 6 – 9, write the following expressions **with** exponents:

<table>
<thead>
<tr>
<th>6) (12 \cdot 12 \cdot 12)</th>
<th>7) ((202)(202)(202)(202))</th>
</tr>
</thead>
<tbody>
<tr>
<td>8) (3 \cdot 3 \cdot 5 \cdot 5 \cdot 5)</td>
<td>9) (2 \cdot 2 \cdot 2 \cdot 15 \cdot 15 \cdot 15 \cdot 15)</td>
</tr>
<tr>
<td>Question</td>
<td>Expression</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
</tr>
<tr>
<td>10)</td>
<td>$4^3 \cdot 2^2 \cdot 5$</td>
</tr>
<tr>
<td>12)</td>
<td>1452</td>
</tr>
<tr>
<td>14)</td>
<td>What is the square of 49?</td>
</tr>
<tr>
<td>16)</td>
<td>Evaluate: $\sqrt{81}$</td>
</tr>
<tr>
<td>18)</td>
<td>What is the square of 625?</td>
</tr>
<tr>
<td>20)</td>
<td>Evaluate: $\sqrt{144}$</td>
</tr>
</tbody>
</table>
For questions 1 – 5, perform the indicated operations (evaluate):

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>$36 \div 4 \times 3 - 4 + 7$</td>
<td></td>
</tr>
<tr>
<td>2)</td>
<td>$\sqrt{81} - (1^8 + 3^2 - 1) + 2^3 \times 7$</td>
<td></td>
</tr>
<tr>
<td>3)</td>
<td>$20 - \sqrt{36} + (17 - \sqrt{16} \times 2)$</td>
<td></td>
</tr>
<tr>
<td>4)</td>
<td>$2^5 \times 6 \div 3 - \sqrt{3^2} + 4^2 + 5\sqrt{121}$</td>
<td></td>
</tr>
<tr>
<td>5)</td>
<td>$\sqrt{\sqrt{(\sqrt{81} - \sqrt{64})^{10}} - 2^3 + \sqrt{169}}$</td>
<td></td>
</tr>
</tbody>
</table>
2.1 extra practice answers:

1) 37 and 5 are the factors, 185 is the product
NOTE: Because of this, 185 is a multiple of 37 and 185 is a multiple of 5.

2) 8007 and 1003 are the factors and 8,031,021 is the product
NOTE: Because of this, 8,031,021 is a multiple of 8007 and 8,031,021 is a multiple of 1003.

3) You always get zero
4) The number itself
5) 1, 2, 4, 5, 10, 20
6) 1, 2, 3, 5, 6, 10, 15, 30
7) 1, 7, 49
8) Associative
9) Identity
10) 398 x 413

2.2 extra practice answers:

1) 17,056
2) 246,214
3) 3,626,718
4) 54,000
5) 600,000
6) 5,600,000
7) (12 x 7) + (12 x 9)
8) 1,600,000
9) 480,000
10) 1,000,000
11) $203,670

2.3 extra practice answers:

1) $\text{x} = 13 \text{ feet}
2) 283 sq ft
3) 196 sq in
4) 158 sq ft

2.4 extra practice answers:

1) 5 x 3 = 15
2) $15 \div 3 = 5$ and $15 \div 5 = 3$
3) $6 \div 2 = 3$
4) $\text{x} = 13$
5) $\text{x} = 8$
6) $\text{x} = 175$
7) $\text{x} = 8000$
8) *
9) no *

2.5 extra practice answers:

1) 802
2) 370
3) 268 r 2
4) 110
5) 28 r 40
6) 66
7) 20
8) 190
9) 11 buses*
10) 143 *

2.6 extra practice answers:

1a) 10 gallons
1b) $40
2) 21 mpg
3a) area = 104 sq. ft. perimeter = 42 feet
3b) $924
4) *

* These Questions & Answers may require some discussion.
2.7 extra practice answers:

1) 420; 42,367,943,520; 90; and 3800 2) yes *

3) see table below

<table>
<thead>
<tr>
<th></th>
<th>4554</th>
<th>1950</th>
<th>33,121</th>
<th>12,133</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>10</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>11</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>13</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

2.8 extra practice answers:

1) 1 * 2) composite numbers * 3) 180=2x2x3x3x5

4) 455=5x7x13 5) 1564=2x2x17x23 6) 19,404=2x2x3x3x7x7x11

2.9 extra practice answers:

1) 2 x 7; 2\(^7\) \(\neq\) 128 2) 5 x 5 x 5 x 5 x 5 3) 4 x 4 x 4 x 4 x 4 x 4

4) 3 x 3 x 3 x 3 x 8 x 8 x 8 x 12 5) 16 x 16 x 16 x 16 x 31 x 31

6) 12\(^3\) 7) 202\(^4\) 8) 3\(^3\) x 5\(^3\) 9) 2\(^2\) x 15\(^4\)

10) 1280 11) 1 12) 2\(^2\) x 3 x 11\(^2\) 13) 3\(^2\) x 7 x 11\(^2\)

14) 2401 15) 7 16) 9 17) 6561

18) 390,625 19) 25 20) 12 21) 20,736

2.10 extra practice answers:

1) 30 2) 56 3) 23 4) 114 5) 4

* These Questions & Answers may require some discussion.
CHAPTER 3

PROBLEM SOLVING

WITH

MULTIPLICATION AND DIVISION

OF

FRACTIONS
Chapter 3.1

FRACTION INTRODUCTION

- People think of a fraction as representing a piece, a part or a portion of one whole thing.

- A slice of cake could be $\frac{1}{12}$ of the cake, $\frac{1}{8}$ of the cake, or if you are really hungry, $\frac{1}{2}$ of the cake.

- Fractions also represent division problems. Recall fraction notation for division from chapter 2: Numerator ÷ Denominator (or Top ÷ Bottom).

Recall that $\frac{6}{2} = 3$, is fraction notation for division. $\frac{6}{2}$ means six halves of one unit (see below). When you put six halves of one unit together you get three whole units. (Think cake!)

\[ \text{NOTE: Halves means 2 equal size pieces make one whole unit.} \]

Fractions and division go hand in hand.

\[ \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \]

\[ 1 \text{ whole unit} \quad + \quad 1 \text{ whole unit} \quad + \quad 1 \text{ whole unit} = 3 \text{ whole units} \]

\[ \text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{the number of pieces wanted or shaded}}{\text{the number of equal size pieces one whole unit is broken into}} \]

What fractions are represented by the shaded portion in the following figures?

Ex. A:

\[ \frac{5}{6} \]

\[ \text{NOTE: All pieces represented in any fraction must be EQUAL in size.} \]
Ex. B:  

[Diagram showing 1 shaded piece out of 2 pieces in one full bar. $\frac{1}{2}$ is the fraction pictured here.]

Ex. C:  

[Diagram showing 2 shaded pieces out of 3 pieces in one full bar. $\frac{2}{3}$ is the fraction pictured here.

Ex. D:  

[Diagram showing 3 shaded pieces out of 2 pieces in one full bar. $\frac{3}{2}$ is the fraction pictured here.]

Ex. E:  

[Diagram showing 7 shaded pieces out of 3 pieces in one full bar. $\frac{7}{3}$ is the fraction pictured here.]

Ex. F:  

[Diagram showing 4 shaded pieces out of 4 pieces in one full bar. $\frac{4}{4}$ is the fraction pictured here. (4 ÷ 4 = 1 whole bar)]

NOTE: The numerator can be smaller than the denominator, larger than the denominator or equal to the denominator.
For the following problems, what fractional part is shaded?

1) \[
\begin{array}{cccc}
& & & \\
\end{array}
\]

2) \[
\begin{array}{cccc}
& & & \\
\end{array}
\]

3) \[
\begin{array}{cccc}
\end{array}
\]

4) \[
\begin{array}{cccc}
\end{array}
\]

5) \[
\begin{array}{cccc}
\end{array}
\]

6) \[
\begin{array}{cccc}
\end{array}
\]
There are 2 kinds of fractions:

1) **PROPER FRACTIONS**: A fraction is considered to be **proper** when the numerator is **smaller than** the denominator.

NOTE: All proper fractions are between 0 and 1 in value and fall between 0 and 1 on the number line.

Examples of Proper Fractions: \(\frac{1}{2}, \frac{2}{5}, \frac{4}{16}, \frac{6}{9}\)

NOTE: When you divide a proper fraction on the calculator (numerator ÷ denominator or top ÷ bottom) you get a quotient that has a whole number part equal to zero followed by a decimal point with one or more digits to the right of the decimal point.

Proper fraction \(\rightarrow\) decimal = 0.(decimal portion)

\[
\frac{1}{2} = 1 \div 2 = 0.5 \\
\frac{2}{5} = 2 \div 5 = 0.4 \\
\frac{4}{16} = 4 \div 16 = 0.25 \\
\frac{6}{9} = 6 \div 9 = 0.66666... 
\]

NOTE: Each answer (quotient) above has a whole number part equal to zero.

2) **IMPROPER FRACTIONS**: A fraction is considered to be **improper** when the numerator is **greater than or equal to** the denominator.

NOTE: All improper fractions have a value that is **greater than or equal to** 1.

Examples of improper fractions: \(\frac{3}{1}, \frac{5}{4}, \frac{17}{5}, \frac{6}{6}\)

NOTE: When you divide an improper fraction (top ÷ bottom) the numerator always **divides by** the denominator at least once.

NOTE: All improper fractions can be changed into **mixed numbers** (or whole numbers if the fraction divides evenly, meaning there is no decimal or remainder).

NOTE: More on changing improper fractions to mixed numbers later.

\[
\frac{3}{1} = 3 \div 1 = 3 \\
\frac{5}{4} = 5 \div 4 = 1.25 \\
\frac{17}{5} = 17 \div 5 = 3.4 \\
\frac{6}{6} = 6 \div 6 = 1 \\
= 1\frac{1}{4} \\
= 3\frac{2}{5} 
\]

NOTE: Each answer (quotient) above has a whole number part equal to 1 or more.
FRACTION PICTURE WORD PROBLEMS

Ex. A: If you and some friends order a pizza pie (8 slices) and 5 slices are eaten, what fraction of a pie was eaten? What fraction of a pie remains?

Five pieces were eaten: five pieces were eaten
eight pieces in one full pie
Written as a fraction, $\frac{5}{8}$ of a pie was eaten.

Three pieces remain: three pieces remain
eight pieces in one full pie
Written as a fraction, $\frac{3}{8}$ of a pie remains.

Ex. B: If you and some friends order two pizza pies (8 slices each) and 13 slices are eaten, what fraction of a pie was eaten? What fraction of a pie remains?

NOTE: Notice the phrasing of the question, “What fraction of a pie?” A pie has only 8 slices. The denominator here is 8, because 1 full unit is one pie with 8 equal size pieces.

Thirteen pieces were eaten: thirteen pieces were eaten
eight pieces in one full pie
Written as a fraction, $\frac{13}{8}$ of a pie was eaten.

Three pieces remain: three pieces remain
eight pieces in one full pie
Written as a fraction, $\frac{3}{8}$ of a pie remains.
This question can be asked differently, for example:

**Ex. C:** If you and some friends order two pizza pies (8 slices each) and 13 slices are eaten, what fraction of the pizza was eaten? What fraction of the pizza remains?

**NOTE:** Notice the phrasing of the question, “What fraction of the pizza?” There are 16 slices of pizza in total. The denominator here is 16, because 1 full unit is all the pizza, which is two pies.

**Answer:** \( \frac{13}{16} \) of the pizza was eaten and \( \frac{3}{16} \) of the pizza remains.

**NOTE:** Here the reference point is different (“the pizza” vs “a pie”). Some information may be lost when phrasing the problem this way.

**NOTE:** Look at the numerators and denominators of Ex. A, Ex. B, and Ex. C. Think about what type of fraction is appropriate in each example: proper or improper.

1) In a class of 33 students there are 12 female students. What fractional part of the class is female?

2) There are 21 female students in two classes combined. If there are 15 students in each class, what fractional part of the two classes is female?
MIXED NUMBERS

A **mixed number** combines of two types of numbers, a **whole number** and a **proper fraction**. Some specific amounts can be represented by either a **mixed number** or an **improper fraction**.

**Ex. A:** If you ate 2 whole candy bars plus $\frac{1}{3}$ of another candy bar, how much candy did you eat? Write your answer as a mixed number and in fractional notation.

To think of this as an improper fraction, the denominator must be the same for each full bar. If each bar is broken into 3 equal size pieces (this says the denominator = 3) then you ate 7 pieces where each piece is $\frac{1}{3}$ of a bar. This it can be written as the **improper fraction** $\frac{7}{3}$. If you use the mixed number, $2 \frac{1}{3}$ bars, or the improper fraction, $\frac{7}{3}$ bars, you ate the same amount of candy.

![Diagram showing mixed number and improper fraction]

**Answer:** You ate $2 \frac{1}{3}$ candy bars which is the same as $\frac{7}{3}$ of a candy bar.

(Whether written as a mixed number or an improper fraction, you ate too much candy!)

**Converting Mixed Numbers to Improper Fractions:**

**Ex. A:** Convert $2 \frac{1}{3}$ to an improper fraction (fraction notation).

**Step 1:** The **denominator** represents the total number of pieces in one whole bar. Here there are 2 whole bars, so multiply the whole number by the denominator $(2 \times 3) = 6$ pieces in 2 whole bars.

**Step 2:** The **numerator** in the proper fraction represents the extra piece(s), so add this numerator to the product you got in Step 1 ($6 + 1$) to get 7 pieces total in the numerator of the improper fraction.

**Step 3:** The **denominator** stays the same for the mixed number and the improper fraction because it is the total number of equal size pieces in 1 whole bar.

**Answer:** $2 \frac{1}{3} = \frac{7}{3}$ as an improper fraction.
Converting Improper Fractions to Mixed Numbers:

Instead of multiplying and adding we will reverse the process and divide and subtract.

\[
\begin{align*}
\frac{3}{3} \text{ bars} & \hspace{1em} + \hspace{1em} \frac{3}{3} \text{ bars} \hspace{1em} + \hspace{1em} \frac{1}{3} \text{ bar} \\
& = \hspace{1em} 2 \text{ whole bars} \hspace{1em} + \hspace{1em} \frac{1}{3} \text{ bar} \\
\frac{7}{3} \text{ bars} & = \hspace{1em} 2 \frac{1}{3} \text{ bars} \\
\text{Improper Fraction} & = \hspace{1em} \text{Mixed Number}
\end{align*}
\]

**Ex. B:** Convert \( \frac{7}{3} \) to a mixed number.

**Step 1:** The denominator represents the total pieces in one whole bar. So, divide the numerator by the denominator to find the number of whole bars in 7 total pieces. \( 7 \div 3 = 2.3333 \ldots \). The whole number is 2 and that is the whole number in the mixed number. **Write the 2 as the whole number.** Ignore the .333… here.

**Step 2:** The denominator stays the same for the mixed number and the improper fraction because it is the total number of pieces in one whole bar so set up your answer by writing the 3 in the denominator of the proper fraction in the mixed number.

**Step 3:** The numerator in the proper fraction part of the mixed number represents the extra piece(s). There were 7 pieces and 6 were used to make 2 whole bars, how many extra pieces are left? Subtract to get \( 7 - 6 = 1 \). This is the 1 extra piece which is the **numerator of the proper fraction** in the mixed number.

**Answer:** \( \frac{7}{3} = 2 \frac{1}{3} \) as a Mixed Number.
Convert the following Mixed Numbers to Fraction Notation (Improper Fractions).

1) \(4 \frac{3}{5}\)  
2) \(2 \frac{5}{6}\)  
3) \(20 \frac{7}{10}\)

Convert the following Improper Fractions to Mixed Numbers or Whole Numbers.

1) \(\frac{11}{4}\)  
2) \(\frac{17}{17}\)  
3) \(\frac{33}{5}\)

NOTE: You can significantly speed up the process of converting improper fractions to mixed numbers by memorizing some decimal fraction equivalents.

<table>
<thead>
<tr>
<th>0.5</th>
<th>(\frac{1}{2})</th>
<th>0.2</th>
<th>(\frac{1}{5})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.333… = 0.(\overline{3})</td>
<td>(\frac{1}{3})</td>
<td>0.4</td>
<td>(\frac{2}{5})</td>
</tr>
<tr>
<td>0.666… = 0.(\overline{6})</td>
<td>(\frac{2}{3})</td>
<td>0.6</td>
<td>(\frac{3}{5})</td>
</tr>
<tr>
<td>0.25</td>
<td>(\frac{1}{4})</td>
<td>0.8</td>
<td>(\frac{4}{5})</td>
</tr>
<tr>
<td>0.75</td>
<td>(\frac{3}{4})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Ex. A:** Convert \(\frac{11}{4}\) to a mixed number.

\(\frac{11}{4} = 11 \div 4 = 2.75 = 2 \frac{3}{4}\) as a mixed number.

Since 0.75 = \(\frac{3}{4}\) we can replace the decimal part of this number with its fractional equivalent.

**Answer:** \(\frac{11}{4} = 2 \frac{3}{4}\) as a mixed number.
Homework 3.1  

Name:  

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Numerator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Denominator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Proper Fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Improper Fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Mixed Number</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For questions 6 – 8, indicate what fractional part is shaded:

6)  

7)  

8)  

For questions 9 – 11, draw a picture to represent the given fraction:

9)  \( \frac{2}{7} \)  

10)  \( \frac{2}{8} \)  

11)  \( \frac{7}{2} \)  

12) Two friends share an 8-slice pizza pie. Each friend eats 2 slices. What fractional part of a pie remains?

13) Five friends buy three 8-slice pizza pies. The first two people have 2 slices each and the remaining people have 3 slices each. What fractional part of a pie remains?
For problems 14 – 19, change the following improper fractions to mixed numbers:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>7/3</td>
<td>15</td>
</tr>
<tr>
<td>17</td>
<td>7/4</td>
<td>18</td>
</tr>
</tbody>
</table>

For problems 20 – 25, change the following mixed numbers to improper fractions:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4 1/3</td>
<td>21</td>
</tr>
<tr>
<td>23</td>
<td>3 1/4</td>
<td>24</td>
</tr>
</tbody>
</table>
Chapter 3.2

RENNAMING FRACTIONS

You can rename fractions using one of two previously learned facts:

**The Identity Property** for multiplication: \( N \times 1 = N \)

**Basic Division Rule:** \( N \div 1 = N \)

That is, the appearance of any fraction can be changed, **without changing its value**, when you **multiply or divide** that fraction by 1.

Multiplying or dividing any fraction by 1, does not change its appearance **UNLESS** you use a different version of the number 1.

**NOTE:** Remember, since \( \frac{N}{N} = 1 \) and \( N \div N = 1 \), you can use any Natural Number for \( N \), except 1.

For example, one candy bar can be split evenly between two children by cutting it into 2 equal pieces. Give \( \frac{1}{2} \) (one piece) to one child and the other \( \frac{1}{2} \) (the other piece) to the second child.

Mathematically it looks like this:

\[
\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{2}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}
\]

One child’s share is \( \frac{1}{2} \) of the bar of candy.

If you ask them if they want 1 piece of candy or 2 pieces, of course they say two but there is only 1 candy bar. Cut their one larger piece into two smaller pieces. Now they each have two pieces of candy.

Mathematically it looks like this:

\[
\frac{1}{2} = \frac{1}{2} \times 1 \text{ but } \frac{2}{2} = 1 \text{ so substitute } \frac{2}{2} \text{ for } 1
\]

**NOTE:** As the denominator increases the size of the pieces get smaller.
Ex. A: Build or create an equal fraction for $\frac{3}{5}$ by multiplying by 1, but use the version: $\frac{N}{N}$ for the number 1. You pick the value for N. There will be different answers for different people.

Using N = 4: $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$

Answer: $\frac{3}{5} = \frac{12}{20}$ as an equal fraction.

Ex. B: Find the missing numerator for $\frac{9}{2} = \frac{A}{14}$.

Method 1: Find the correct version of $1 = \frac{N}{N}$ used to multiply $\frac{9}{2}$ to get $\frac{A}{14}$.

$$\frac{9}{2} \times \frac{N}{N} = \frac{A}{14} \Rightarrow 9 \times N = \frac{A \times 14}{2} \Rightarrow 9 \times 7 = \frac{63}{14}$$

N = 7, so multiplying both the numerator and denominator by 7 (because $2 \times 7 = 14$ in the denominator), we get $9 \times 7 = 63$ in the numerator.

Method 2: Cross multiply.

$$\frac{9}{2} = \frac{A}{14} \Rightarrow 2 \times A = 9 \times 14 \Rightarrow 2A = 126 \Rightarrow A = 63$$

To Cross Multiply you must find two cross products: Multiply the numerator of one fraction by the denominator of the other fraction. Do this diagonally in both directions. Then set the two cross products equal to each other.

Answer: The missing numerator is 63.

Find the missing numerator or denominator in each of the following problems (solve for A).

1) $\frac{3}{2} = \frac{18}{A}$

2) $\frac{5}{8} = \frac{30}{A}$

3) $\frac{1}{4} = \frac{A}{32}$

4) $\frac{6}{10} = \frac{A}{15}$
TESTING FOR EQUAL FRACTIONS

Once you rename a fraction, it is a good idea to test and see IF the given fraction is equal (equivalent) to the new, renamed fraction.

Two fractions are equal if their cross-products are equal.

We used cross multiplying in Method 2, Ex. B of the previous exercise.

The Fraction Equality Rule:

Given two equal fractions, cross multiplication results in two equal cross products.

Use this rule when there are two fractions with an equal sign between them.

\[
\frac{N}{D} = \frac{n}{d}
\]

There is only one thing you can do with these two fractions: cross multiply them.

\[
N x d = D x n
\]

NOTE: You ONLY cross multiply when you see an equal sign between two fractions!

**Ex. A:** Is \(\frac{5}{11} = \frac{4}{7}\)? (Are the two fractions equivalent?)

There is only one thing you can do when there is an equal sign between two fractions, cross multiply.

When you cross multiply you get cross products.

Ask yourself, are the cross products equal (is 35 = 44)? No, of course not. Since the cross products are not equal, then the two fractions are not equal (\(\frac{5}{11}\) is not equal to \(\frac{4}{7}\)).

**Answer:** \(\frac{5}{11} \neq \frac{4}{7}\) because the cross products are not equal.

NOTE: The mathematical symbol, “≠” represents the words “is not equal to.”
Are the following pairs of numbers equal?

1) \( \frac{3}{8} = \frac{6}{16} \) ?

2) \( \frac{8}{10} = \frac{12}{15} \) ?

3) \( \frac{3}{4} = \frac{9}{16} \) ?

4) \( \frac{36}{72} = \frac{10}{20} \) ?

5) \( \frac{4}{5} = \frac{5}{6} \) ?

6) \( \frac{17}{14} = \frac{51}{42} \) ?

(HINT: Questions 7 and 8 are not fractions so ask “What should I do first?”)

7) \( 2 \frac{3}{5} = 4 \frac{6}{10} \) ?

8) \( 5 \frac{3}{4} = 5 \frac{39}{52} \) ?
REDUCING FRACTIONS

Fractional answers **MUST** always be expressed in *lowest terms*. If not, there would be an infinite number of equivalent fractional answers. All would be correct but it would create confusion. The rule is to present all fractional answers in *lowest terms*.

What does it mean to have a fraction in *lowest terms*?

Definition: **Lowest Terms**: A fraction is in *lowest terms* if the numerator and denominator have NO factors in common (other than 1).

**Method 1: Reducing with algebra:**

1) **Factor**: Write the numerator and the denominator as a product of prime factors.

2) **Cancel**: Look for ones as matching pairs of prime factors above below (Note: \( \frac{K}{K} = 1 \))

3) **Multiply**: Multiply any remaining primes

Ex. A: Reduce \( \frac{12}{36} \) to Lowest Terms.

\[
\begin{align*}
\frac{12}{36} & = \frac{2 \times 2 \times 3}{2 \times 2 \times 3 \times 3} = \frac{1 \times 1 \times 1}{1 \times 1 \times 1 \times 3} = \frac{1}{3} \\
\end{align*}
\]

**Step 1:**
- Write the prime factorization of 12
- Write the prime factorization of 36

**Step 2:**
- Cancel pairs of matching factors (1 for 1)
- (look for 1s)
- Remember: \( \frac{n}{n} = 1 \)

**Step 3:**
- Multiply remaining factors

**Step 4:**
- Always cross multiply to check your answer. If the cross products are equal then your answer is equivalent to the given fraction. Be sure the numerator and denominator have **no factors in common** to be certain your answer is in *lowest terms*.

Answer: \( \frac{12}{36} \) reduced to lowest terms is \( \frac{1}{3} \).
**Method 2: Reducing by dividing numerator and denominator by a common factor.**

This is similar to building fractions, but here we use the basic division fact: \( \frac{N}{N} = \frac{N}{N} \), where 1 is expressed as \( \frac{N}{N} \) to reduce fractions.

**Ex. A:** (Method 2) Reduce \( \frac{12}{36} \) to Lowest Terms.

\[
\begin{align*}
\frac{12}{36} & \div \frac{2}{2} = \frac{6}{18} \\
\frac{6}{18} & \div \frac{2}{2} = \frac{3}{9} \\
\frac{3}{9} & \div \frac{3}{3} = \frac{1}{3}
\end{align*}
\]

- **Step 1:** 2 is a common factor for both 12 and 36.
  \[
  \frac{12}{2} = 6 \\
  \frac{36}{2} = 18
  \]
  \[
  \text{Note: } \frac{2}{2} = 1 \quad \text{and} \quad \frac{N}{D} \div 1 = \frac{N}{D}
  \]

- **Step 2:** 2 is a common factor for both 6 and 18.
  \[
  \frac{6}{2} = 3 \\
  \frac{18}{2} = 9
  \]
  \[
  \text{Note: } \frac{3}{3} = 1 \quad \text{and} \quad \frac{N}{D} \div 1 = \frac{N}{D}
  \]

- **Step 3:** 3 is a common factor for both 3 and 9.
  \[
  \frac{3}{3} = 1 \\
  \frac{9}{3} = 3
  \]

**Answer:** \( \frac{12}{36} \) reduced to lowest terms is \( \frac{1}{3} \)

**Note:** Cross multiplying to check your answer won’t always work in this method. If you missed a common factor, the cross products will still be equal and the fractions will be equivalent, but your answer will not be in lowest terms and it will be incorrect. This method works well for students who are strong in their times tables facts and remember to consider all possible factors.

**Method 3: Old School:**

Use this if you are a strong math student. It’s quick and easy, but you run the risk of missing a common factor or writing the incorrect factor. If you want to use this method, BE CAREFUL!

\[
\begin{align*}
1 & \\
2 & \\
6 & \div \frac{12}{36} = \frac{1}{3} \quad \text{(Look familiar?)}
\end{align*}
\]

(This is because most people do the division in their heads and just write down the quotients.)

\[
\begin{align*}
48 & \\
6 & \\
3 & \div \frac{48}{6} = \frac{3}{3}
\end{align*}
\]

**Answer:** \( \frac{12}{36} \) reduced to lowest terms is \( \frac{1}{3} \)

**NOTE:** Method I: Reducing with Algebra has two important advantages.

First, it is the most accurate method whether you are a strong or weak math student.

Second, this method will also be used to make fraction multiplication and division fairly simple.
Reduce the following to lowest terms using Method 1 and check your answers (see page 219).

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Reduced Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{6}{30} )</td>
<td>( \frac{1}{5} )</td>
</tr>
<tr>
<td>( \frac{16}{40} )</td>
<td>( \frac{2}{5} )</td>
</tr>
<tr>
<td>( \frac{15}{20} )</td>
<td>( \frac{3}{4} )</td>
</tr>
<tr>
<td>( \frac{39}{66} )</td>
<td>( \frac{13}{22} )</td>
</tr>
<tr>
<td>( \frac{8}{48} )</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>( \frac{126}{90} )</td>
<td>( \frac{7}{5} )</td>
</tr>
<tr>
<td>( \frac{77}{56} )</td>
<td>( \frac{11}{8} )</td>
</tr>
<tr>
<td>( \frac{440}{990} )</td>
<td>( \frac{4}{9} )</td>
</tr>
<tr>
<td>( \frac{54}{90} )</td>
<td>( \frac{3}{5} )</td>
</tr>
<tr>
<td>( \frac{702}{1170} )</td>
<td>( \frac{11}{19} )</td>
</tr>
</tbody>
</table>

**A look ahead or When will I ever need this?**

Method 1 is algebra. Reducing numerical fractions with algebraic methods will help you practice the same method you will use to simplify complicated fractions in algebra.
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
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</thead>
<tbody>
<tr>
<td>1) Numerator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Denominator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Equivalent Fractions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Cross Products</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Proportion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Test for Equal Fractions (Cross Multiply)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For questions 7 – 10, solve for \( n \):

<table>
<thead>
<tr>
<th>Question</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>7)</td>
<td>( \frac{n}{3} = \frac{6}{9} )</td>
</tr>
<tr>
<td>8)</td>
<td>( \frac{16}{n} = \frac{4}{5} )</td>
</tr>
<tr>
<td>9)</td>
<td>( \frac{6}{15} = \frac{n}{35} )</td>
</tr>
<tr>
<td>10)</td>
<td>( \frac{91}{119} = \frac{13}{n} )</td>
</tr>
</tbody>
</table>

For questions 11 – 14, test for equality (test for equal fractions):

<table>
<thead>
<tr>
<th>Question</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>11)</td>
<td>( \frac{25}{36} = \frac{5}{6} ) ?</td>
</tr>
<tr>
<td>12)</td>
<td>( \frac{3}{4} = \frac{6}{7} ) ?</td>
</tr>
<tr>
<td>13)</td>
<td>( \frac{76}{95} = \frac{4}{5} ) ?</td>
</tr>
<tr>
<td>14)</td>
<td>( \frac{5}{8} = \frac{40}{72} ) ?</td>
</tr>
</tbody>
</table>

For questions 15 – 18, reduce the fractions to lowest terms (simplify completely):

<table>
<thead>
<tr>
<th>Question</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>15)</td>
<td>( \frac{35}{45} )</td>
</tr>
<tr>
<td>16)</td>
<td>( \frac{15}{9} )</td>
</tr>
<tr>
<td>17)</td>
<td>( \frac{6}{30} )</td>
</tr>
<tr>
<td>18)</td>
<td>( \frac{1050}{700} )</td>
</tr>
</tbody>
</table>
19) **Challenge problem**: reduce the following fraction to lowest terms (simplify completely):

\[
\frac{6552}{11,088}
\]

**Challenge problems with algebra**:

Reduce the following fractions to lowest terms (simplify completely):

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>( \frac{x^2}{x^5} )</td>
<td>( \frac{2x^5}{4x^2} )</td>
</tr>
<tr>
<td>2)</td>
<td>( \frac{12x^4y^5}{21x^4y^8} )</td>
<td>( \frac{81x^3y^5z}{54x^4y^3z^3} )</td>
</tr>
</tbody>
</table>
CHAPTER 3.3

FRACTION MULTIPLICATION

We use fraction multiplication all the time and aren’t even aware of it. If you have $4 and are going to share it equally with your brother or sister you will each get \( \frac{1}{2} \) of the $4, and using common sense (and our need to ensure our brother or sister does not get a bigger share) we know instinctively that you and your sibling will each get $2.

But when some are asked the question, “What is \( \frac{1}{2} \) of 4?” the appearance of numbers as fractions tends to make some of us forget common sense.

When asked the question, “What is \( \frac{1}{2} \) of \( \frac{1}{4} \)?” that can be more confusing. If you remember that \( \frac{1}{2} \) of anything means to break that thing into 2 equal pieces, you can draw a picture to get a visual image of the problem.

**Ex. A:** What is \( \frac{1}{2} \) of \( \frac{1}{4} \)?

First draw a picture of a “cake” and cut it into 4 equal pieces. One piece is \( \frac{1}{4} \) of the cake, so we have a picture of \( \frac{1}{4} \).

Cut one of the pieces in \( \frac{1}{2} \) making two (smaller) equal size pieces. What is the size of one of the two smaller pieces?

To find the size of the new smaller piece, cut the remaining three pieces in \( \frac{1}{2} \) as well. The fraction represented by one of the new smaller pieces is \( \frac{1}{8} \).

**Answer:** \( \frac{1}{2} \) of \( \frac{1}{4} \) is \( \frac{1}{8} \).

**Note:** This makes sense. If you cut a slice of cake in \( \frac{1}{2} \), you should get a smaller piece and \( \frac{1}{8} \) of a cake is smaller than \( \frac{1}{4} \) of a cake.
The Fraction Multiplication Rule

What happens when the fractions become larger and it is too difficult to draw pictures? Then we rely on the rules for multiplying fractions.

Follow this rule when you see two (or more) fractions with a multiplication sign between them:

- **Stretch** the fraction line across the two fractions:
  \[
  \frac{N \times n}{D \times d} = \frac{N \times n}{D \times d}
  \]

- **Factor:**
  - Find a prime factorization for all the numerators
  - Find a prime factorization for all the denominators

- **Cancel:**
  - Cancel pairs of matching primes, 1 for 1, above and below:
    - Look for 1s: \( \frac{n}{n} = 1 \)

- **Multiply:**
  - Multiply the remaining primes in the numerator
  - Multiply the remaining primes in the denominator

- **Convert:**
  - Ask yourself, “Is the answer a proper fraction?”
    - If it is, stop, you are finished.
    - If it isn’t, then convert the improper fraction into a mixed number, to become familiar with alternate forms of the answer (and for word problems), using the method on page 209.

**Ex A:** Find the product of \( \frac{1}{2} \) and \( \frac{1}{4} \).

\[
\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1 \times 1}{2 \times 2 \times 2} = \frac{1 \times 1}{2 \times 2 \times 2} = \frac{1}{8} = \frac{\text{small}}{\text{large}}
\]

(No canceling) (No converting)

<table>
<thead>
<tr>
<th><strong>STRETCH</strong></th>
<th><strong>FACTOR</strong></th>
<th><strong>CANCEL</strong></th>
<th><strong>MULTIPLY</strong></th>
<th><strong>CONVERT</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>the line between the two fractions.</td>
<td>both the numerator and the denominator into their prime factors.</td>
<td>all pairs of matching factors above and below, 1 for 1. Look for 1s: ( \frac{n}{n} = 1 )</td>
<td>the remaining numerators and remaining denominators separately on top and bottom.</td>
<td>Is the answer a proper fraction? Yes, STOP.</td>
</tr>
</tbody>
</table>

**Answer:** The product of \( \frac{1}{2} \) and \( \frac{1}{4} \) is \( \frac{1}{8} \).
Find the following products.

1) \( \frac{2}{3} \times \frac{5}{7} = \)

2) \( \frac{14}{15} \times \frac{10}{7} = \)

3) \( \frac{18}{25} \times \frac{10}{12} = \)

3) \( \frac{21}{33} \times \frac{15}{28} = \)

Ex. B: Find the product of \( \frac{3}{5} \) and 10.

\[
\frac{3}{5} \times 10 = \frac{3 \times 10}{5 \times 1} = \frac{3 \times 2 \times 5}{5 \times 1} = \frac{1}{1}
\]

**Product** means to multiply.

All numbers MUST be in fraction form, \( 10 = \frac{10}{1} \)

**STRETCH** the line between the two fractions.

**FACTOR** both the numerator and the denominator into their prime factors.

**CANCEL** all pairs of matching factors above and below, 1 for 1.

Look for 1s: \( \frac{n}{n} = 1 \)

**MULTIPLY** the remaining numerators and the remaining denominators.

\[
\frac{3 \times 2 \times 1}{1 \times 1} = \frac{6}{1} = 6 \div 1 = 6
\]

**CONVERT** Is the answer a proper fraction? No, convert to a whole number since the numerator divides evenly by the denominator.

Answer: The product of \( \frac{3}{5} \) and 10 is 6.

**NOTE:** In this problem, 10 is a whole number and the rules for fractions only work if every number in the problem is in fraction form.
Ex. C: \( \frac{2}{21} \times 2 \frac{1}{4} \times \frac{7}{18} = ? \)

\[
\begin{array}{cccc}
& \frac{2}{21} & \times & \frac{9}{4} \times \frac{7}{18} \\
\text{STRETCH} & \frac{2 \times 9 \times 7}{21 \times 4 \times 18} & \text{FACtor} & \frac{2 \times 3 \times 3 \times 7}{3 \times 7 \times 2 \times 2 \times 2 \times 3 \times 3} \\
\text{CANCEL} & \frac{1 \times 1 \times 1}{1 \times 1 \times 2 \times 2 \times 1 \times 3} & = & \frac{1}{12} \\
\end{array}
\]

Numbers MUST be in fraction form, \( \frac{2}{4} = \frac{9}{4} \).

\( \frac{1\times 1 \times 1 \times 1}{1 \times 1 \times 2 \times 2 \times 1 \times 3} \)

Answer: \( \frac{1}{12} \) is the product of \( \frac{2}{21} \times 2 \frac{1}{4} \times \frac{7}{18} \).

Find the following products.

1) \( 12 \times \frac{3}{4} = \)

2) \( 1\frac{1}{6} \times \frac{3}{22} \times \frac{11}{14} = \)

3) \( 2\frac{2}{5} \times 3\frac{1}{3} = \)

4) \( 4\frac{1}{5} \times 1\frac{11}{14} \times \frac{5}{9} = \)
Estimating with Mixed Numbers:

It is always a good idea to do a quick estimate to see if your answer makes sense. When estimating with mixed numbers you can simply ignore the proper fraction and work with the whole number part of the mixed number.

**Ex. A:** Estimate and find the exact product: \(3\frac{1}{2} \times 6\frac{3}{4}\)

The estimate for this problem would be \(3 \times 6 = 18\).

**NOTE:** When the whole number part of the mixed number is a single digit, you can simply “chop off” (truncate) the fraction part of the mixed number. Because the fraction part usually represents a very small part of the mixed number, it is generally not a big deal to just “toss it.”

**NOTE:** When the whole number part of the mixed number has two or more digits, you would “toss” the fraction part and do the estimate as you would with simple whole numbers. In this case the proper fraction part would be a rather insignificant part of the mixed number.

**NOTE:** To estimate \(3\frac{1}{2} \times 6\frac{3}{4}\) using more accurate rounding, you would get \(4 \times 7 = 28\). Using this and the previous estimate, the exact answer should be somewhere between 18 and 28.

To find the exact product: \(3\frac{1}{2} \times 6\frac{3}{4} = \frac{7}{2} \times \frac{27}{4} = \frac{189}{8} = 23\frac{5}{8}\)

**Answer:** The estimated product of \(3\frac{1}{2} \times 6\frac{3}{4}\) is 18. The exact product of \(3\frac{1}{2} \times 6\frac{3}{4}\) is \(23\frac{5}{8}\).

Estimate the following products:

1) \(12\frac{3}{4} \times 6\frac{3}{4}\)
2) \(1\frac{1}{6} \times 2\frac{3}{22} \times 1\frac{11}{14}\)
3) \(2\frac{2}{5} \times 3\frac{1}{3}\)
4) \(4\frac{1}{5} \times 1\frac{11}{14} \times 5\frac{5}{9}\)

Mixed Numbers MUST be changed to improper fractions. No reducing is possible, so multiply numerators and then multiply denominators. Change the improper fraction back to a mixed number (because the problem started with mixed numbers).
COMMUTATIVE AND ASSOCIATIVE LAWS APPLY TO FRACTIONS

Commutative Law:  
\[
\frac{1}{3} \times \frac{4}{5} = \frac{4}{5} \times \frac{1}{3}
\]
\[
\frac{1 \times 4}{3 \times 5} = \frac{4 \times 1}{5 \times 3}
\]
\[
\frac{4}{15} = \frac{4}{15}
\]

Associative Law:  
\[
\left(\frac{1}{2} \times \frac{3}{4}\right) \times \frac{5}{7} = \frac{1}{2} \times \left(\frac{3}{4} \times \frac{5}{7}\right)
\]
\[
\frac{(1 \times 3) \times 5}{(2 \times 4) \times 7} = \frac{1 \times (3 \times 5)}{2 \times (4 \times 7)}
\]
\[
\frac{3 \times 5}{8 \times 7} = \frac{1 \times 15}{2 \times 28}
\]
\[
\frac{15}{56} = \frac{15}{56}
\]

NOTE: Once the rules for multiplying fractions are applied, the commutative or associative law is applied to both the numerator and the denominator.

SQUARES AND SQUARE ROOTS OF FRACTIONS AND MIXED NUMBERS

\[
(Fraction)^2 = \left(\frac{\text{Numerator}}{\text{Denominator}}\right)^2 = \frac{\text{Numerator}^2}{\text{Denominator}^2}
\]

and

\[
\sqrt{\text{Fraction}} = \sqrt{\frac{\text{Numerator}}{\text{Denominator}}} = \frac{\sqrt{\text{Numerator}}}{\sqrt{\text{Denominator}}}
\]

**Ex. A:** Find the square of \(\frac{4}{9}\).

The square of \(\frac{4}{9}\) is \(\left(\frac{4}{9}\right)^2 = \frac{4^2}{9^2} = \frac{16}{81}\)

**Ex. B:** Find the square root of \(\frac{4}{25}\).

The square root of \(\frac{4}{25}\) is \(\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{\sqrt{2^2}}{\sqrt{5^2}} = \frac{2}{5}\)

**Ex. C:** Find the square of \(1\frac{2}{3}\).

The square of \(1\frac{2}{3}\) is \(\left(1\frac{2}{3}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9} = 2\frac{7}{9}\)

**Ex. D:** Find the square root of \(1\frac{7}{9}\).

The square root of \(1\frac{7}{9}\) is \(\sqrt{1\frac{7}{9}} = \sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{\sqrt{4^2}}{\sqrt{3^2}} = \frac{4}{3} = 1\frac{1}{3}\)

NOTE: We use the rules of fraction multiplication to find squares and square roots. Since \(1\frac{2}{3}\) in **Ex. C** and \(1\frac{7}{9}\) in **Ex. D** are mixed numbers, we need to convert them to improper fractions **first** and then apply fractions rules. Since the answers we get are improper fractions, they need to be converted back to mixed numbers. **Remember, you generally end with the same form of a number that you had at the start of a problem.** Here we started with mixed numbers, so end with mixed numbers, if possible.
Evaluate the following:

1) \( \left( \frac{2}{5} \right)^2 \) 
2) \( \left( 1 \frac{2}{3} \right)^2 \)

3) \( \left( \frac{3}{8} \right)^2 \) 
4) \( \left( 2 \frac{1}{4} \right)^2 \)

5) \( \sqrt{\frac{9}{25}} \) 
6) \( \sqrt{\frac{9}{16}} \)

7) \( \sqrt{\frac{9}{36}} \) 
8) \( \sqrt{\frac{5}{25}} \)
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Numerator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Denominator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Product</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Square</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Square Root</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For questions 7 – 8, Use pictures to show the following:

7) Show $\frac{1}{2}$ of 4:

8) Show $\frac{1}{2}$ of $\frac{1}{4}$:

9) Complete the following statement using the Commutative Law:
$$\frac{1}{2} \times \frac{3}{4} =$$

For questions 10 – 17, find the following products:

<table>
<thead>
<tr>
<th>10)</th>
<th>11)</th>
<th>12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{11} \times \frac{2}{5}$</td>
<td>$\frac{3}{11} \times \frac{11}{17}$</td>
<td>$\frac{10}{42} \times \frac{21}{18}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>13)</th>
<th>14)</th>
<th>15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{4} \times \frac{2}{15} \times \frac{5}{9}$</td>
<td>$4 \times 1\frac{1}{2}$</td>
<td>$1\frac{1}{2} \times 2\frac{2}{3}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>16)</th>
<th>17)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{30}{21} \times \frac{13}{12} \times \frac{14}{39}$</td>
<td>$3\frac{1}{2} \times \frac{5}{7} \times \frac{8}{15}$</td>
</tr>
</tbody>
</table>

18) Challenge Problem: **Find the following product:**
$$\frac{48}{105} \times \frac{42}{63} \times \frac{15}{8}$$
For questions 19 – 30, evaluate the following expressions:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>19)</td>
<td>$\left(\frac{16}{25}\right)^2$</td>
<td>20)</td>
</tr>
<tr>
<td>22)</td>
<td>$\sqrt{\frac{16}{25}}$</td>
<td>23)</td>
</tr>
<tr>
<td>25)</td>
<td>Find the square of $\frac{4}{9}$</td>
<td>26)</td>
</tr>
<tr>
<td>28)</td>
<td>Find the square of $\frac{49}{81}$</td>
<td>29)</td>
</tr>
</tbody>
</table>
CHAPTER 3.4

DIVISION OF FRACTIONS

Def. Reciprocal: Two numbers of any type (fractions, whole numbers, decimals, percents) are reciprocals if their product is 1.

To FIND the reciprocal of any number:
First convert the number to fraction form then “flip” the fraction.

\[
\begin{align*}
\text{Given number} & \quad \text{Its reciprocal} & \quad \text{(given number) x (its reciprocal)} \\
\frac{1}{2} &\quad \frac{2}{1} &\quad \frac{1}{2} \times \frac{2}{1} = \frac{1 \times 2}{2 \times 1} = \frac{2}{2} = 1 \\
\frac{2}{3} &\quad \frac{3}{2} &\quad \frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1 \\
1\frac{2}{3} &\quad 1\frac{2}{3} = \frac{5}{3} &\quad \frac{5}{3} \times \frac{3}{5} = \frac{5 \times 3}{3 \times 5} = \frac{15}{15} = 1 \\
7 &\quad 7 = \frac{7}{1} &\quad \frac{7}{1} \times \frac{1}{7} = \frac{7 \times 1}{1 \times 7} = \frac{7}{7} = 1 \\
0 &\quad 0 = \frac{0}{1} &\quad \frac{1}{0} \rightarrow \text{undefined} &\quad 0 \text{ has no reciprocal.}
\end{align*}
\]

NOTE: It makes sense that 0 has no reciprocal if you look at the definition of reciprocals. 0 x anything always equals 0. One can never multiply any number by 0 to get a product of 1.

The Fraction Division Rule:
Follow this rule when you see two fractions with a division sign between them:

Have \( \frac{\text{fraction}}{\div \frac{\text{fraction}}{}} \) ? Change the division sign to multiplication and multiply by the reciprocal of the \( \text{2nd} \) fraction.

\[
\frac{\begin{align*}
N &\quad n \\
D &\quad d
\end{align*}}{\div \frac{\begin{align*}
N &\quad d \\
D &\quad n
\end{align*}}{}} = \frac{\begin{align*}
N &\quad n \\
D &\quad d
\end{align*}}{\times \frac{\begin{align*}
N &\quad d \\
D &\quad n
\end{align*}}{}}
\]

OR when you see: \( \text{fraction} \div \frac{\text{fraction}}{\} \), it’s simpler to think:

\[
\frac{\begin{align*}
N &\quad n \\
D &\quad d
\end{align*}}{K \iff T \iff F} = \frac{\begin{align*}
N &\quad d \\
D &\quad n
\end{align*}}{K \iff T \iff F}
\]

Step 1) \text{K} eep the first fraction.
Step 2) \text{C} hange the division sign to a multiplication (\text{T}imes) sign.
Step 3) \text{F} lip the \text{2nd} fraction. (NOTE: only flip the \text{2nd} fraction, and only with division!!)
NOTE: If you see division of fractions: change the division to multiplication by keeping the first fraction the same and multiplying by the reciprocal of the 2nd fraction. Often referred to as:

Keep → Times → Flip

or

Keep → Change → Flip

or

Keep → Change → Change

Why?...

Look at the following two problems.

1) \( \frac{10}{1} \div \frac{2}{1} = \frac{10}{1} \times \frac{1}{2} = \frac{10 \times 1}{1 \times 2} = \frac{10}{2} = \frac{5}{1} = 5 \)

2) \( \frac{1}{2} \times 10 = 10 \times \frac{1}{2} = \frac{10}{1} \times \frac{1}{2} = \frac{10 \times 1}{1 \times 2} = \frac{10}{2} = \frac{5}{1} = 5 \)

This first step bring us back to the same multiplication as above, so continue as above to get 5 as the final answer.

If you use common sense, \( \frac{1}{2} \times 10 \) means “half of ten” in word form. Half of ten is 5.

10 ÷ 2 means “split ten into two equal size pieces.” Each resulting piece is 5.
Another explanation for this fraction division uses complex fractions:

Complex fractions, fractions within other fractions, are not so nice to look at.

They can be “cleaned up” by doing the following:

1) Turn the denominator, which is a fraction, into a 1 by multiplying it by its reciprocal.

2) Multiply the numerator, which is also a fraction, by the exact same number used in step 1.

3) Now we have a multiplication problem on top and a 1 on the bottom of this complex fraction. Since anything ÷ 1 is “itself, we are really just left with the multiplication problem on top.

4) Finish the multiplication problem as before.

If you follow the basic rule for dividing fractions you get:

\[
\frac{\frac{2}{3}}{\frac{4}{9}} = \frac{2}{3} \div \frac{4}{9} = \frac{2 \cdot 9}{3 \cdot 4} = \frac{2 \cdot 3 \cdot 3}{3 \cdot 2 \cdot 2} = \frac{3}{2}
\]

Answer: \( \frac{\frac{2}{3}}{\frac{4}{9}} = \frac{2}{3} \div \frac{4}{9} = \frac{3}{2} \)
Ex. A: Find the quotient of \( \frac{2}{5} \) and \( \frac{4}{11} \)

\[
\frac{2}{5} \div \frac{4}{11} = \frac{2}{5} \times \frac{11}{4} = \frac{2 \times 11}{5 \times 4} = \frac{2 \times 11}{5 \times 2 \times 2} = \]

**Quotient means to divide**

- **Keep the 1st fraction**
- **Times:** \( \div \) becomes \( \times \)
- **Flip the 2nd fraction**

**STRETCH** the line between the two fractions.

**FACTOR** the numerators and the denominators into prime factors.

\[
\frac{\frac{2 \times 11}{5 \times 2 \times 2}}{\frac{1}{1 \times 1 \times 2}} = \frac{\frac{1 \times 11}{5 \times 1 \times 2}}{\frac{11}{10}} = 1 \frac{1}{10}
\]

**CANCEL** any matching factors above and below, 1 for 1. Look for 1s:
\[
\frac{n}{n} = 1
\]

**MULTIPLY** the remaining numerators and then the denominators.

**CONVERT:** Is the answer a proper fraction? No, convert it to a mixed number to become familiar with equivalent forms of the answer (and for word problems).

**Answer:** The quotient of \( \frac{2}{5} \) and \( \frac{4}{11} \) is \( 1 \frac{1}{10} \).

Find the following quotients.

1) \( \frac{5}{18} \div \frac{10}{6} = \)

2) \( \frac{18}{5} \div \frac{3}{10} = \)

3) \( \frac{25}{40} \div \frac{5}{8} = \)

4) \( \frac{5}{4} \div \frac{25}{28} = \)
**Ex. B:** Find the quotient of $1 \frac{1}{4}$ and 15.

$$
1 \frac{1}{4} \div 15 = \frac{5}{4} \div \frac{15}{1} = \frac{5}{4} \times \frac{1}{15} = \frac{5 \times 1}{4 \times 15} = \frac{5 \times 1}{2 \times 2 \times 3 \times 5}
$$

<table>
<thead>
<tr>
<th>Quotient means to divide.</th>
<th>Numbers <strong>MUST</strong> be in fraction form first.</th>
<th><strong>K T F</strong> Keep the 1st fraction <strong>T</strong>imes: $\div$ becomes $\times$ <strong>F</strong>lip the 2nd fraction</th>
<th><strong>STRETCH</strong> the line between the two fractions.</th>
<th><strong>FACTOR</strong> the numerators and the denominators into prime factors.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{5}{4} \div \frac{15}{1}$</td>
<td>$\frac{5}{4} \times \frac{1}{15}$</td>
<td>$\frac{5 \times 1}{4 \times 15}$</td>
<td>$\frac{5 \times 1}{2 \times 2 \times 3 \times 5}$</td>
<td></td>
</tr>
</tbody>
</table>

**CANCEL** any matching factors, above and below, $1$ for $1$: $\frac{n}{n} = 1$

<table>
<thead>
<tr>
<th><strong>MULTIPLY</strong> the remaining numerators and then the denominators.</th>
<th><strong>CONVERT</strong>: Is the answer a proper fraction? Yes, stop.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{5 \times 1}{2 \times 2 \times 3 \times 5}$</td>
<td>$\frac{1}{12}$</td>
</tr>
</tbody>
</table>

**Answer:** The quotient of $1 \frac{1}{4}$ and 15 is $\frac{1}{12}$

Find the following quotients.

1) $3 \frac{3}{5} \div 3 =

2) $4 \frac{1}{2} \div 3 \frac{3}{8} =

3) $1 \frac{1}{2} \div 1 \frac{2}{3} =

4) $8 \frac{2}{5} \div 2 \frac{1}{10} =$
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

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<th>Official Definition</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Quotient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Reciprocal</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3) Why is zero the only number without a reciprocal?

For questions 4 – 7, find the reciprocals of the following numbers:

<table>
<thead>
<tr>
<th>4) 8</th>
<th>5) 1/3</th>
<th>6) 1</th>
<th>7) 4 $ \frac{1}{2}$</th>
</tr>
</thead>
</table>

8) What should you do first when you see mixed numbers?
9) What is the rule for division involving fractions?

For questions 10 – 17, find the following quotients and simplify:

<table>
<thead>
<tr>
<th></th>
<th>10) ( \frac{3}{5} \div 3 )</th>
<th>11) ( 3 \div \frac{3}{5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12) ( \frac{14}{9} \div \frac{7}{3} )</td>
<td>13) ( \frac{7}{3} \div \frac{14}{9} )</td>
</tr>
<tr>
<td></td>
<td>14) ( 4 \div 1 \frac{1}{2} )</td>
<td>15) ( 1 \frac{1}{2} \div 4 )</td>
</tr>
<tr>
<td></td>
<td>16) ( 4 \div 2 \frac{2}{3} )</td>
<td>17) ( 4 \frac{1}{2} \div 2 \frac{2}{3} )</td>
</tr>
</tbody>
</table>
CHAPTER 3.5

SOLVING EQUATIONS WITH FRACTIONS

Method 1: Using Division to Solve for N.

One way to solve a seemingly difficult problem is by changing it to something “simpler but similar.” “Simpler but similar” is an old math trick. If the rule works for a simpler but similar problem, then it will work for the more difficult problem.

What does a simpler but similar problem look like given the following? \( \frac{2}{3} N = 18 \), solve for N.

Change the original problem \( \frac{2}{3} N = 18 \) to a “simpler but similar” problem, \( 2N = 18 \).

\[
\begin{align*}
2N &= 18 \\
\frac{1}{2} N &= \frac{18}{2} \\
N &= 9
\end{align*}
\]

The same method will work for the original problem, \( \frac{2}{3} N = 18 \). Divide both sides of the equation by the number attached to N by multiplication.

Solve \( \frac{2}{3} N = 18 \) using the same method used to solve \( 2N = 18 \).

\[
\begin{align*}
\frac{2}{3} N &= 18 \\
\frac{2}{3} N &= 18 \div \frac{2}{3} \\
N &= \frac{18}{\frac{2}{3}} \\
N &= \frac{18 \times \frac{3}{2}}{1} \\
N &= \frac{18 \times 3}{2} \\
N &= 27
\end{align*}
\]
Answer: N = 27, so \( \frac{2}{3} \times 27 = 18 \)

**Method 2: Using Multiplication by a Reciprocal to Solve for N.**

How do you solve the following problem?

\[
\frac{2}{3} N = 18
\]

The goal is to solve for N. The problem should simplify to:

N = some number (a whole number or a fraction).

Remember: Any fraction times its reciprocal equals 1 and \(1N = N\).

If you multiply \( \frac{2}{3} \) by its reciprocal you get \( \frac{2}{3} \times \frac{3}{2} \) and this product is 1.

You can multiply an equation by any number as long as you multiply both sides of the equation by the same number to keep the equation balanced.

Write all numbers in fraction form.
STRETCH the line between the fractions on both sides of the equation.

\[
\frac{3 \times 2 \times N}{2 \times 3 \times 1} = \frac{18 \times 3}{1 \times 2}
\]

\[
\frac{3 \times 2 \times N}{2 \times 3 \times 1} = \frac{2 \times 3 \times 3 \times 3}{1 \times 2}
\]

CANCEL pairs of matching factors, 1 for 1, above and below. Look for 1s: \( \frac{K}{K} = 1 \)

\[
\frac{\frac{2 \times 3 \times 3 \times 3}{1 \times 2}}{\frac{2 \times 3 \times 3 \times 3}{1 \times 2}} = \frac{18}{1}
\]

MULTIPLY the remaining numerators and then the denominators.

\[
N = \frac{27}{1}
\]

CONVERT the improper fractions to whole numbers because the numerators divide evenly by the denominators.

N = 27

Answer: N = 27, so \( \frac{2}{3} \times 27 = 18 \).

Method 3: Creating a Proportion to Solve for N.

Write all numbers in fraction form.

\[
\frac{2}{3} \times \frac{N}{1} = \frac{18}{1}
\]

Stretch then multiply the numerators and then the denominators.

\[
\frac{2 \times N}{3 \times 1} = \frac{18}{1}
\]

What can you do when you see two equal fractions? (Fraction = Fraction) Cross multiply.

\[
\frac{2N}{3} = \frac{18}{1}
\]

Divide both sides of the equation by 2 to get N alone.

\[
2N \times 1 = 3 \times 18
\]

\[
2N = 54 \rightarrow \frac{2N}{2} = \frac{54}{2} \rightarrow N = 27
\]

Answer: N = 27, so \( \frac{2}{3} \times 27 = 18 \).

Remember to check the answer, regardless of which method you used to solve the problem!

1st rewrite the problem with ( ) in for N, 2nd substitute the value for N in the ( ), then

DO THE MATH to see if the answer works! \( \frac{2}{3} \times (27) = 18 \), so: \( \frac{2}{3} \times \frac{27}{1} = \frac{2 \times 27}{3 \times 1} = \frac{54}{3} = \frac{18}{1} \)
Solve the following for N:  
(Remember that 2N means 2 \times N or 2 “times” N)

1) \( \frac{3}{8} N = 18 \)  
2) \( N = \frac{3}{7} \times 21 \) 

3) \( 28 = 3 \frac{1}{2} N \)  
4) \( \frac{3}{8} N = \frac{1}{6} \) 

5) \( \frac{2}{3} \times \frac{21}{22} = N \)  
6) \( N \times \frac{3}{20} = 27 \) 

7) (Challenge Problem)  
\( N = 2 \frac{1}{3} \times 6 \frac{3}{7} \) 

8) (Challenge Problem)  
\( N \times \frac{4}{15} = \frac{1}{20} \)
SOLVING SIMPLE ALGEBRAIC WORD PROBLEMS

The simplest word problems can cause great difficulty. One tends to just see the numbers and add, subtract, multiply, or divide or in the case of fractions, cross multiply. A multiple choice test usually has all of those answers as options. Odds are 1 out of 4 you will get the correct answer and that gives you a possible average of 25% on tests. That is unacceptable. Taking a couple of seconds to read the problem carefully and work the problem can save a lot of time in the end.

NOTE: Just because you have an answer doesn’t mean it’s the RIGHT answer!

Ex. A: \( \frac{2}{3} \) of what number is 18?

\[
\frac{2}{3} \text{ of what number is } 18? \quad \text{This problem is written in half Math and half English, “Mathlish.” We translate to all Math.}
\]

1) Circle the numbers \( \frac{2}{3} \) and 18.
2) Circle the words “is” and “of.”
3) Circle whatever is left.
You should have 5 circles.

Translate the numbers and words in the order they are written.
1) \( \frac{2}{3} \) and 18 stay the same.
2) “of” means multiply, “is” means equals
3) “what number” means an unknown quantity

Now the problem to solve is:

\[
\frac{2}{3} \times N = 18 \quad \text{OR} \quad \frac{2}{3} N = 18
\]

Once the problem is written with all numbers and math symbols there are different methods to solve the equation. It is good to become familiar with all three methods.

Method 1: Using division to solve for N.

Method 2: Using multiplication by the reciprocal to solve for N.

Method 3: Creating a proportion to solve for N.

Choose the method that works best for you!
Translate the following problems into simple algebraic equations and solve them.

1) \( \frac{3}{8} \) of what number is 18? 
2) What number is \( \frac{3}{7} \) of 21?

3) 28 is \( 3 \frac{1}{2} \) of what number? 
4) \( \frac{3}{8} \) of what number is \( \frac{1}{6} \)?

5) \( \frac{2}{3} \) of \( \frac{21}{22} \) is what number? 
6) What number times \( \frac{3}{20} \) is 27?

7) (Challenge Problem) 
What fractional part of \( \frac{4}{15} \) is \( \frac{1}{20} \)?

8) (Challenge Problem) 
What is \( 2 \frac{1}{3} \) of \( 6 \frac{3}{7} \)?
Review the rules for fractions below.

Remember, it depends on what you see between the fractions:

1) Fraction Multiplication:
If you see a multiplication sign between two (or more) fractions:

\[
\frac{N}{D} \times \frac{n}{d} = \frac{N \times n}{D \times d}
\]

What can you do?

Stretch the line then factor, cancel & multiply (aka simplify).

2. Fraction Division:
If you see a division sign between two fractions:

\[
\frac{N}{D} \div \frac{n}{d} = \frac{N}{D} \times \frac{d}{n}
\]

What can you do?

Keep Change Flip

DO NOT CANCEL!!!!!!
(At least not yet – wait until you are following multiplication rules.)

3) Fraction Equality Rule:
If you see an equal sign between two fractions:

\[
\frac{N}{D} = \frac{n}{d}
\]

\[
N \times d = D \times n
\]

What can you do? Only one thing, CROSS MULTIPLY!

Caution: NO CANCELING ALLOWED! This is NOT multiplication!
Homework 3.5  Name:  

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Reciprocal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Mixed Number</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 4 – 19, solve and check the following:

4) \[7N = 14\]

5) \[7N = 84\]

6) \[\frac{2}{3} N = 84\]

7) \[\frac{2}{7} N = \frac{1}{14}\]
8) \( \frac{2}{9} N = \frac{14}{3} \)

9) \( \frac{2}{9} N = \frac{9}{14} \)

10) \( 2 \frac{2}{3} N = 48 \)

11) \( 4 \frac{1}{2} N = 24 \)
<table>
<thead>
<tr>
<th></th>
<th>Question</th>
<th></th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>What is ( \frac{1}{2} ) of 4?</td>
<td>13</td>
<td>What is ( \frac{1}{2} ) of ( \frac{1}{4} )?</td>
</tr>
<tr>
<td>14</td>
<td>What is ( \frac{2}{3} ) of ( \frac{1}{6} )?</td>
<td>15</td>
<td>What number is ( \frac{7}{12} ) of 84?</td>
</tr>
</tbody>
</table>
16) What is $\frac{2}{3}$ of 6?

17) $\frac{2}{3}$ of what number is 12?

18) $\frac{1}{5}$ of what number is $\frac{1}{13}$?

19) What fractional part of $\frac{3}{11}$ is $\frac{1}{22}$?
Chapter 3.6

FRACTION OF A WHOLE WORD PROBLEMS

Some problems are very specifically fraction problems. Part of a group has one characteristic while the remainder of the group does not.

There are different methods to solve these types of problems.

- **Method 1:** Translation Method
- **Method 2:** Fraction Picture Method
- **Method 3:** Proportion Method

**Method 1: Translation Method**

**Ex. A:** Half of the class (a fraction or part of the class) has perfect lab attendance. If there are 28 students in the class how many have perfect lab attendance?

The Translation Method is based on finding the key phrase or question and translating from “Mathlish” to all Math, as well as using the numerical information in the correct place.

Finding the phrase “fraction of a whole” is extremely helpful when solving problems of this type with the translation method.

The key phrase in this problem is:
1) Half of the class has perfect lab attendance.

   \[ \text{Fraction of the whole class} \]

Determine the numerical information and what it represents. You will need:

1) The fraction in the problem (these are generally proper fractions).
2) The value of the whole thing.
3) The value of the part of that whole thing.

**NOTE:** You will be given either the value for the whole thing or the value for the part. The other will be the unknown quantity that you must find to answer the question.
Ex. A: Half of the class (a fraction or part of the class) has perfect lab attendance. If there are 28 students in the class how many have perfect lab attendance?

From the key phrase in this problem we can extract the numerical information:
1) Half of the class has perfect lab attendance.

The numerical information and what it represents is:
1) “half” is a word name for \( \frac{1}{2} \)
2) 28 students are in the whole class
3) N is the unknown quantity that represents the number of students with perfect lab attendance. (This is only part of the class).

Extra Information:
1) “of” means to multiply
2) “is” means equal to but there is no word “is” in this problem. Here, the word “has” is standing in for the word “is.”

Sometimes you need to rephrase the problem to see what word is standing in for the word “is.” Here you can read the problem as “Half of the class is the group of students (the part of the class) that has perfect lab attendance.”

\[
\text{Half} \quad \frac{1}{2} \quad \text{of} \quad \text{the class} \quad \times \quad 28 \quad = \quad N
\]

Now the word problem has been “translated” to math. The solution is:

\[
\frac{1}{2} \times 28 = N \quad \rightarrow \quad \frac{1}{2} \times \frac{28}{1} = N \quad \rightarrow \quad \frac{1 \times 28}{2 \times 1} = N \quad \rightarrow \quad \frac{1 \times 2 \times 7}{1 \times 2} = N \quad \rightarrow \quad \frac{14}{1} = N \quad \rightarrow \quad \boxed{14 = N}
\]

N is the number of students with perfect lab attendance and \( N = 14 \) means there are 14 students with perfect lab attendance. There are 28 students in the whole class, so the answer makes sense.

Answer: 14 students in the class have perfect attendance.

NOTE: This is a good method for students with strong reading comprehension.
Ex. B: Two-fifths of my students (a fraction or part of my students) have a lab grade of 90 or higher. If 8 students have a lab grade of 90 or higher, how many students are there in my class?

Translation Method is based on finding the key sentence or question and translating from “Mathlish” to all Math, as well as using the numerical information in the correct place.

The key sentence in this problem is:
1) Two-fifths of my students (a fraction or part of my students) have a lab grade of 90 or higher.

Fraction of the whole class (all the students)

The numerical information is:
1) Two-fifths is a word name for $\frac{2}{5}$
2) 8 students have a lab grade of 90 or higher. (This is part of the class; some of the students.)
3) N is the unknown replacing the total number of students in the class. (Notice the question.)

Extra Information:
1) “of” means to multiply
2) “is” means equal to but there is no word “is” in this problem. However, the word “have” is standing in for the word “is.”

Sometimes you need to rephrase the problem to see what word is standing in for the word “is.” Here you can read the problem as “Two-fifths of my students is the group of students (the part of the class) that has a lab grade of 90 or higher.”

Now the word problem has been “translated” to math. The solution is:

$$\frac{2}{5} \times N = 8 \quad \rightarrow \quad \frac{2}{5} \times \frac{N}{1} = \frac{8}{1} \quad \rightarrow \quad \frac{2 \times N}{5 \times 1} = \frac{8}{1} \quad \rightarrow \quad \frac{2N}{5} = \frac{8}{1} \quad \rightarrow$$

$$2N = 40 \quad \rightarrow \quad \frac{2N}{2} = \frac{40}{2} \quad \rightarrow \quad N = 20$$

N is the number of total students in the class and N = 20 means there are 20 students total in the class. The answer makes sense.

Answer: There are 20 students in the whole class.

NOTE: 2 things:
1) The word “is” does not always appear in the wording of the problem.
2) Sometimes the “of” is just an English word and does not mean multiplication.
**Method 2: Fraction Picture Method**

With this method find the basic phrase “fraction of whole is part” in the problem. If necessary, rephrase the problem to fit this phrase. It is essential to know if you were given the “whole” and are looking for the “part” or you were given the “part” and are looking for the “whole.”

NOTE: You will be given either the value for the whole thing or the part of that whole thing. The other will be the unknown quantity that you must find to answer the question.

Problems like this can be “seen” in a picture form. It is possible to solve problems involving statements such as “part of the class” in an “easy” way. This involves using a picture of the fraction involved and bringing the problem into a whole number world.

1) **Draw a picture** of the fraction given in the problem and label the information provided about the “part” and the “whole,” either as a given number or an unknown quantity.

NOTE: The next two steps follow methods presented in Chapter 2

2) Find “how much is in each equal box” by dividing as follows:

   \[
   \text{Given quantity} \div \text{Number of boxes it fills (represents)} = \text{Amount in each equal box}
   \]

3) **Multiply** to find the answer:

   \[
   \text{Amount in each equal box} \times \text{Number of boxes needed} = \text{Answer}
   \]

As with the Translation Method, it is necessary to determine the numerical information and what it represents. Finding the phrase “fraction of a whole” is helpful in determining this information.

You will need the following numerical information:

1) The **fraction** in the problem.

2) The value of the **whole** thing.

3) The value of the **part** of that whole thing.

NOTE: Only use this method with proper fractions.
**Ex. A:** Half of the class (a fraction or part of the class) has perfect lab attendance. If there are 28 students in the class how many have perfect lab attendance?

Start with “Fraction of whole is part”

Half of the class has perfect lab attendance.

The numerical information is as follows:

1) The fraction in the problem (these are generally proper fractions). Here, the fraction = \( \frac{1}{2} \)

2) The value of the whole thing. Here, we are given 28 students = the whole class.

3) The value of the part of that whole thing. This is the unknown quantity in this problem.

---

**Step 1:** Draw the fraction and label the “part” and label the “whole.”

**Step 2:** There are 28 students in the whole class, so divide the students evenly among both boxes. 

\[ \frac{28}{2} = 14 \text{ students in each box.} \]

**Step 3:** The “part” of the class is the unknown quantity. This is represented by the numerator, so we only need the quantity in one box for the answer.

**Answer:** 14 students have perfect lab attendance.

---

**NOTE:** Proper labeling of the boxes (Step 1) is key to this process.
**Ex. B:** Two-fifths of my students (a fraction or part of my whole class) have a lab grade of 90 or higher. If 8 students have a lab grade of 90 or higher, how many students are there in my class?

Two-fifths \( \frac{2}{5} \) is a fraction that means 2 groups (boxes) out of 5 total groups (boxes) represent the students that have a lab grade of 90 or higher. (See the picture below.)

Start with “Fraction of whole is part”

Two-fifths of my students have a lab grade of 90 or higher.

The numerical information is as follows:

1) The fraction in the problem. Here, the fraction = \( \frac{2}{5} \)

2) The value of the whole thing. This is the unknown quantity in this problem.

3) The value of the part of that whole thing. Here, we are given 8 students = part of the class.

**Step 1:** Draw the fraction and label the “part” and label the “whole.”

![Fraction Picture Diagram]

**Step 1:**

- **This is a picture of the given fraction “two-fifths”** \( \frac{2}{5} \)
- **8 students with a lab grade of 90 or higher.** (This is given)

**Step 2:** There are 8 students in the part of the class with a lab grade of 90 or higher, so divide the students evenly among these two boxes.

\[
\frac{8}{2} = 4 \text{ students in each box}
\]

**Step 3:** The “whole” class is the unknown quantity. This is represented by the denominator, so we need the quantity in all 5 boxes for the answer:

\[
4 \times 5 = 20 \text{ students in the whole class}
\]

**Answer:** There are 20 students in the whole class.

**NOTE:** You will often find that the fraction-picture method will bring problems like this into the whole number world, but sometimes the numbers don’t work out so nicely. This is why it is good to be familiar with several methods to solve these kinds of problems.
Method 3: Proportion Method.

Ex. A: Half of the class (a fraction or part of the class) has perfect lab attendance. If there are 28 students in the whole class how many have perfect lab attendance?

The numerical information is:
1) Half (\( \frac{1}{2} \)) of the class (a fraction or part of the class) has perfect lab attendance. This means that in a group of 2 students in the class 1 of them has perfect lab attendance.
2) 28 students are in the whole class (total number of students).
3) \( N \) is the number of students with perfect attendance. Notice that the question asks, “How many students have perfect attendance?”

The first ratio for the proportion method is:

\[
\frac{1 \text{ student has perfect lab attendance}}{2 \text{ students total}}
\]

Once the first ratio is established, **WITH LABELS**, the second ratio can be written with the labels first, **then** filling in the numbers.

Match labels on the top!

\[
\frac{1 \text{ student has perfect lab attendance}}{2 \text{ students total}} = \frac{N \text{ students with perfect attendance}}{28 \text{ students in the whole class}}
\]

Match labels on the bottom!

Matching labels is necessary to set up the proportion correctly. Once this setup is complete, remove the labels, keeping the numbers where they are, to get the following equation to solve:

\[
\frac{1}{2} = \frac{N}{28} \quad \rightarrow \quad 2N = 28 \quad \rightarrow \quad \frac{2N}{2} = \frac{28}{2} \quad \rightarrow \quad N = 14
\]

But, \( N = 14 \) what? Go back to the original problem to find the correct label for \( N \):

\[
\frac{1 \text{ student has perfect lab attendance}}{2 \text{ students total}} = \frac{N=14 \text{ students with perfect attendance}}{28 \text{ students in the whole class}}
\]

Answer: 14 students in the class have perfect attendance.

NOTE: The Proportion Method relies heavily on the use of word labels to describe the numbers.
Ex. B: Two-fifths of my students (a fraction or part of my students) have a lab grade of 90 or higher. If 8 students have a lab grade of 90 or higher, how many students are there in my class?

The numerical information is:
1) Two-fifths \( \left( \frac{2}{5} \right) \) of the class (a fraction or part of the class) have a lab grade of 90 or higher, This means in a group of 5 students, 2 of them have a lab grade of 90 or higher.
2) 8 students have a lab grade of 90 or higher.
3) \( N \) is the total number of students in the whole class. Notice the question, “How many students are there in my class?”

The first ratio for the proportion method is:

\[
\frac{2 \text{ students have a lab grade of 90 or higher}}{5 \text{ students total}}
\]

Once the first ratio is established, **WITH LABELS**, the second ratio can be written with the labels first, then filling in the numbers.

\[
\frac{2 \text{ students with a lab grade of 90 or higher}}{5 \text{ students total}} = \frac{\square \text{ students with a lab grade of 90 or higher}}{\square \text{ students in the whole class}}
\]

\[
\frac{2 \text{ students with a lab grade of 90 or higher}}{5 \text{ students total}} = \frac{8 \text{ students with a lab grade of 90 or higher}}{N \text{ total number of students in the class}}
\]

As before, matching the labels is essential for setting up the proportion correctly. Once this is done, remove the labels and keep the numbers where they are to get the following problem:

\[
\frac{2}{5} = \frac{8}{N} \quad \rightarrow \quad 2N = 40 \quad \rightarrow \quad \frac{2N}{2} = \frac{40}{2} \quad \rightarrow \quad N = 20
\]

Answer: There are 20 students in the class.

**NOTE:** The Proportion Method relies heavily on the use of word labels to describe the numbers.
Method 3: Another way to look at the Proportion Method:

Start by rephrasing the problem (if necessary). Remember you are looking for the phrase “fraction of a whole is a part.” Drawing and labeling the picture can help.

Knowing if you were given the “whole” and are looking for the “part” or you were given the “part” and are looking for the “whole” is essential.

Because these problems most often involve proper fractions you can set up a simple proportion based on basic fraction knowledge of what a proper fraction represents:

\[
\begin{array}{c}
\text{Proper} \\
\text{Fraction} \\
\end{array} = \begin{array}{c}
\text{Part} \\
\text{Whole} \\
\end{array}
\]

1) Write the fraction given in the problem as the first ratio in the proportion.

2) Write an equal sign and a fraction line for the 2nd ratio in the proportion.

3) Determine what represents the part & the whole, either as a given number or an unknown quantity. The phrase “fraction of a whole is a part” will help you figure this out.

4) Write the value for the “part” in the numerator and the value for the “whole” in the denominator. One will be a given number and the other will be an unknown quantity. Remember to include labels! Once the proportion is correctly set up, you are ready to solve for N.

**Ex. A:** Half of the class (a fraction or part of the class) has perfect lab attendance. If there are 28 students in the class how many have perfect lab attendance?

Start with “Fraction of whole is part”

\[
\frac{1}{2} = \frac{N}{28}
\]

Answer: There are 14 students in the class.

NOTE: Knowing if you were given the “whole” and are looking for the “part” or if you were given the “part” and are looking for the “whole” is essential.
**Ex. B:** Two-fifths of my students (a fraction or part of my students) have a lab grade of 90 or higher. If 8 students have a lab grade of 90 or higher, how many students are there in my class?

Start with “Fraction of whole is part”

Two-fifths of my students have a lab grade of 90 or higher.

Remember that sometimes you need to rephrase the problem to see what word is standing in for the word “is.” Here you can read the problem as “Two-fifths of my students is the group of students (the part of the class) that has a lab grade of 90 or higher.”

Start with “Fraction of whole is part”

Two-fifths of my students is the group of students that has a lab grade of 90 or higher.

The numerical information is:
1) Two-fifths \(\left(\frac{2}{5}\right)\) of the class (a fraction or part of the class) have a lab grade of 90 or higher.
2) 8 students have a lab grade of 90 or higher. This is part of the class.
3) \(N\) is the total number of students in the class. Notice the question, “How many students are there in my class?”

\[
\frac{2}{5} = \frac{8}{N}
\]

\[
2N = 40 
\]

\[
\frac{2N}{2} = \frac{40}{2} 
\]

\[
N = 20
\]

Answer: There are 20 students in the class.

**NOTE:** The previous three methods are all a little different yet they all are similar with regard to the information that the problems provide. To make sense of these problems you must consider the wording. Just realizing that the problem involves a “fraction of a whole is part of that thing” helps you identify the type of problem you were given. You must think about the part and the whole. You must consider what information was provided in the problem. You must consider what the problem is asking you to find. These methods can help you become better at solving word problems. Just remember that it is beneficial to become familiar with all of them.
Solve the following problems using any of the methods described above.

1) In a class of 48 students \( \frac{3}{8} \) of the class had an average of 80 on the first two tests. If there are 48 students in the class, how many of them had an average of 80 on the first two tests?

2) If you earn $55 for working 1 day and you only work \( \frac{4}{5} \) of a day, how much will you earn?

3) If a specialty loaf of bread weighs \( 3 \frac{1}{2} \) lbs, what does \( \frac{1}{2} \) of a loaf weigh?

4) The number of BEP students in 2016 is now \( 3 \frac{1}{8} \) times as large as it was in 1990. There are now 600 students in the program, how many students were in the program in 1990?
FRACTION RATE PROBLEMS

Rate word problems are when you are given numbers for two of the following, “the value for one thing (value per item),” “the total number of things (items),” or “the total value” and you are asked to solve for the missing number.

Method 1: Translation Method
Method 2: Proportion Method

Method 1: Translation Method uses the labels of the numbers to create an algebraic equation that you will then solve for the unknown.

Recall the formula from Chapter 2, page 145:

_value for one thing_ \times _total number of things_ = _total value_

or

_value per item_ \times _number of items_ = _total value_

Ex. A: Linoleum tile costs \( \frac{1}{4} \) dollars per square foot. If you need 120 square feet for the floor in the kitchen, how much will it cost to replace the linoleum in the kitchen?

The numerical information is:
1) \( \frac{1}{4} \) dollars per square foot is the cost for 1 square foot of linoleum, “the value for one thing.”
2) 120 square feet is the total amount of square feet needed; it is “the total number of things.”
3) N dollars, the missing number, is the cost for all the linoleum; it is “the total value.”

\[
\frac{1}{4} \text{ dollars per square foot} \times 120 \text{ square feet} = N \text{ dollars}
\]

Removing all words and writing the numbers and math symbols, (“\times” and “=”) in the order they are written, the problem becomes:

\[
\frac{1}{4} \times 120 \rightarrow \frac{5}{4} \times \frac{120}{1} \rightarrow \frac{5 \times 120}{4 \times 1} \rightarrow \frac{5 \times 2 \times 2 \times 1 \times 5}{2 \times 2 \times 1} \rightarrow \frac{150}{1} \rightarrow 150 = N
\]

Answer: It will cost $150 to replace the linoleum in the kitchen.

NOTE: Does the answer “$150 to replace linoleum” make sense?
Yes, because linoleum is inexpensive and the kitchen floor is fairly small.
**Ex. B:** Gasoline costs \(2 \frac{1}{4}\) dollars per gallon. If you spent 36 dollars to fill up your car with gas, how many gallons of gas does your car hold?

The numerical information is:
1) \(2 \frac{1}{4}\) dollars per gallon is the cost for 1 gallon of gas; it is “the value for one thing.”
2) 36 dollars is the cost for all of the gas; it is “the total value.”
3) \(N\) gallons is the total number of gallons, the unknown; it is “the total number of things.”

Removing all words and writing just the numbers and math symbols, (“\(x\),” \(N\), and “=”) in the order they are written, the problem becomes:

\[
2 \frac{1}{4} \text{ dollars per gallon} \times N \text{ gallons} = 36 \text{ dollars}
\]

Once the problem is written with all numbers and math symbols there are different methods to solve the equation. Use one of the following methods to solve for \(N\) (see pages 247 – 249).

**Method 1:** Using division to solve for \(N\).
**Method 2:** Using multiplication by the reciprocal to solve for \(N\).
**Method 3:** Creating a proportion to solve for \(N\).

\[
2 \frac{1}{4} \times N = 36 \quad \rightarrow \quad N = 16 \text{ gallons}
\]

Answer: Your car holds 16 gallons of gas.

NOTE: Does the answer 16 gallons make sense?
Yes, because most cars have gas tanks that hold between 10 and 20 gallons of gas.
**Method 2: Proportion Method** creates a proportion and solves for the unknown by cross multiplication.

**Ex. A:** Linoleum tile costs $1 \frac{1}{4}$ dollars per square foot. If you need 120 square feet for the floor in the kitchen, how much will it cost to replace the linoleum in the kitchen?

The numerical information is:

1) $1 \frac{1}{4}$ dollars per square foot forms the first ratio: \( \frac{1\frac{1}{4}}{1} \) dollars per square foot
2) 120 square feet is the total amount of square feet of linoleum needed.
3) N dollars, the missing number, is the cost for all of the linoleum.

\[ \frac{1\frac{1}{4}}{1 \text{ square foot}} \] is the first ratio.

Once the first ratio is established, **WITH LABELS**, the second ratio can be written with the labels first, then filling in the numbers.

Now, removing the labels and keeping the numbers where they are, the problem becomes:

\[ \frac{1\frac{1}{4}}{1} = \frac{N}{120} \]

Now what? What is \( 1\frac{1}{4} \) equal to?

Recall: \( \frac{\text{Anything}}{1} = \text{Anything} \) \( \rightarrow \frac{1\frac{1}{4}}{1} = 1 \frac{1}{4} = \frac{5}{4} \)

The problem is now easy to solve:

\[ \frac{\frac{5}{4}}{120} \rightarrow 4N = 600 \rightarrow \frac{4N}{4} = \frac{600}{4} \rightarrow N = 150 \]

**Answer:** It will cost $150 to replace the linoleum in the kitchen.
**Ex. B:** Gasoline costs $2{1\over 4}$ dollars per gallon. If you spent $36$ for gas to fill up your car, how many gallons of gas does your car hold?

The numerical information is:
1) $2{1\over 4}$ dollars per gallon is the cost for 1 gallon of gas and it forms the first ratio.
2) $36$ is the cost for all of the gas.
3) $N$ gallons is the total number of gallons, which is the unknown.

$${2{1\over 4} \text{ dollars} \over 1 \text{ gallon of gas}} \text{ is the first ratio.}$$

Once the first ratio is established, **WITH LABELS**, the second ratio can be written with the labels first, **then** filling in the numbers.

Now, removing the labels and keeping the numbers where they are, the problem becomes:

$$\frac{2{1\over 4} \text{ dollars}}{1 \text{ gallon of gas}} = \frac{36 \text{ total dollars}}{N \text{ total gallons of gas}}$$

Recall: $\frac{\text{Anything}}{1} = \text{Anything} \rightarrow \frac{2{1\over 4}}{1} = 2\frac{1}{4} = \frac{9}{4}$

The problem is now easy to solve:

$$\frac{9}{4} = \frac{36}{N} \rightarrow 9N = 144 \rightarrow \frac{9N}{9} = \frac{144}{9} \rightarrow N = 16$$

Answer: Your car holds 16 gallons of gas.

**NOTE:** All the fraction rules work only for fractions, not mixed numbers but that is an easy fix. Always convert all mixed numbers to improper fractions, then all the fraction rules will apply.
Solve the following Rate Problems.

1) On a map 1 inch represents 42 miles. How many miles does $\frac{3}{7}$ in. represent?

2) A project for an art class requires 20 pieces of ribbon where each piece of ribbon must be $\frac{3}{4}$ ft long. How many total feet of ribbon should be purchased so that every student in the class will have enough?

3) Some fishing poles require a piece of fishing line that is $33 \frac{1}{3}$ yards long for each pole. How many of these fishing poles can be set up from a spool of fishing line that is 300 yards long?

4) The math lab directors are planning to buy calculators that cost $14\frac{1}{2}$ each. If the budget for calculators is $100, how many calculators can be purchased?
CUTTING OR SEPARATING WORD PROBLEMS

There is an exception to “Rate” word problems. If you catch some key words you can make your life very easy. These problems are called “cutting” or “sharing” something where all the pieces or portions are equal in size.

These can be solved with simple division as follows:

1) Ask yourself, “What is being cut or shared,” this will be the dividend.

2) After you cut (or share) something you get pieces (or portions), so the divisor is the information about the pieces (or portions). This information will be given. It is either the total number of pieces (or portions) or the size of each piece (or portion).

3) The quotient is the other information about the pieces, the unknown.

| 1st ask “What is being cut or shared?” | ÷ | Given information about the pieces | = | The other information about the pieces |

Ex. A: A piece of wire \( \frac{3}{5} \) m long is being cut into 15 equal size pieces. How long is each piece?

1) What is being cut?
The piece of wire (which is \( \frac{3}{5} \) m long).

2) “cut into” means “÷”
Given information about the pieces: 15 pieces.

3) N, the quotient, is the other information about the pieces: The length of each piece.

The wire is being cut into 15 pieces

The problem is now a fairly simple fraction division problem:

\[
\frac{3}{5} \div 15 \rightarrow \frac{3}{5} \div \frac{15}{1} \rightarrow \frac{3}{5} \times \frac{1}{15} \rightarrow \frac{3}{5 \times 15} \rightarrow \frac{3}{5} \times \frac{1}{3} \times \frac{5}{15} \rightarrow \frac{2}{5} \times \frac{1}{x} \times \frac{5}{1} \xrightarrow{\text{cancel}} \frac{1}{25} \rightarrow \frac{1}{25} = N
\]

Answer: Each piece is \( \frac{1}{25} \) meters long.

NOTE: Does \( N = \frac{1}{25} \) make sense as an answer? Should the answer be 25, and not \( \frac{1}{25} \)?
The length of the wire being cut is only \( \frac{3}{5} \) meters long and the pieces have to be smaller than the starting length. There is no way you can get 15 pieces of wire that are each 25 meters long from a piece of wire that is only \( \frac{3}{5} \) meters long. Your answer is correct.
**Ex. B:** A piece of wire 15 meters long is being cut into equal size pieces that are each \( \frac{3}{5} \) meter long. How many pieces can be cut?

<table>
<thead>
<tr>
<th>1) What is being cut?</th>
<th>“cut into” means “÷”</th>
<th>2) Given information about the pieces: each piece is ( \frac{3}{5} ) m</th>
<th>3) ( N ), the quotient, is the other information about the pieces: The number of pieces</th>
</tr>
</thead>
<tbody>
<tr>
<td>The wire</td>
<td>is being cut into</td>
<td>pieces</td>
<td>=</td>
</tr>
<tr>
<td>15</td>
<td>÷</td>
<td>( \frac{3}{5} )</td>
<td>( N )</td>
</tr>
</tbody>
</table>

The problem is now a fairly simple fraction division problem:

\[
15 \div \frac{3}{5} = \frac{15 \times 5}{1 \times \frac{3}{5}} = \frac{75}{\frac{3}{5}} = \frac{75 \times 5}{3} = \frac{375}{3} = 125 = N
\]

**Answer:** 25 pieces can be cut.

**NOTE:** Does “\( N = 25 \)” make sense as an answer? The length of the wire being cut is 15 meters long. Each of the pieces is \( \frac{3}{5} \) of a meter which is less than 1 meter long, so there must be at least 15 pieces. The answer, “25 pieces can be cut,” is reasonable.

**NOTE OF CAUTION:**

The Word “cut” does **not** always mean divide.
“Cutting” means division when all the pieces are exactly the same size.
“Cutting” is subtraction when the pieces are **not** all the same size.
Solve the following problems.

1) A rope that is 21 yards long is cut into pieces that are $\frac{3}{7}$ yards long each. How many pieces can be cut?

2) If a 20 pound wedge of cheese is cut into servings weighing $\frac{2}{5}$ pound each, how many servings can be cut?

3) A piece of wire that is $\frac{3}{4}$ of a meter long is to be cut into smaller pieces that are each $\frac{1}{12}$ of a meter long. How many pieces can be cut?

4) A length of string that is $\frac{3}{16}$ yard long is to be cut into 9 equal sized pieces. How long is each piece?
AREA PROBLEMS WITH FRACTIONS

The formula for the area of a rectangle is: Area = Length x Width. This formula works regardless of the types of numbers used for the length and width. The Area formula works when the length and width are whole numbers, fractions, mixed numbers and/or decimals.

Ex. A: Find the area of a rectangle that is $\frac{5}{8}$ yard long by $\frac{4}{5}$ yard wide.

The numerical information is:
1) The length is $\frac{5}{8}$ yards.
2) The width is $\frac{4}{5}$ yards.
3) The formula for finding area of a rectangle is: $A = L \times W$, where $L$ stands for the length, $W$ stands for the width, and $A$ stands for the Area.

\[
\begin{align*}
\text{Area} & = \text{Length} \times \text{Width} \\
A & = L \times W \\
A & = \frac{5}{8} \text{yd} \times \frac{4}{5} \text{yd}
\end{align*}
\]

Remove the labels and keep the numbers where they are to get:

\[
A = \frac{5}{8} \times \frac{4}{5} \rightarrow A = \frac{5 \times 4}{8 \times 5} \rightarrow A = \frac{5 \times 2 \times 2}{2 \times 2 \times 2 \times 5} \rightarrow
\]

\[
A = \frac{\frac{5 \times 2 \times 2}{2 \times 2 \times 2}}{2} \rightarrow A = \frac{1 \times 1 \times 1}{1 \times 1 \times 2 \times 1} \rightarrow A = \frac{1}{2}
\]

Finally, the label for the Area of a rectangle (or any shape) is always square units. The unit of measurement for this problem is yards so the label for the Area must be square yards.

Answer: The area of a rectangle $\frac{5}{8}$ yard long by $\frac{4}{5}$ yard wide is $\frac{1}{2}$ sq yd (square yard) or $\frac{1}{2}$ yd$^2$.

NOTE: The formula for area does not depend on the type of numbers used for length and width.
**Ex. B:** Find the area of a rectangle that is $1 \frac{1}{3}$ yd long by $4 \frac{1}{2}$ yd wide.

The numerical information is:

1) $\text{Area} = L \times W$
   
2) $L = 1 \frac{1}{3}$ yds

3) $W = 4 \frac{1}{2}$ yds

Remove the labels and keep the numbers where they are to get:

\[
\text{Area} = \frac{4}{3} \times \frac{9}{2} \rightarrow \text{Area} = \frac{4 \times 9}{3 \times 2} \rightarrow \text{Area} = \frac{2 \times 2 \times 3 \times 3}{2 \times 2} \rightarrow \text{Area} = \frac{2 \times 1 \times 1 \times 3}{1 \times 1} \rightarrow \text{Area} = \frac{6}{1} \rightarrow \text{Area} = 6
\]

Finally, the label for the Area of a rectangle (or any shape) is always square units. The unit of measurement for this problem is yards so the label for the Area must be square yards.

**Answer:** The area of a rectangle $1 \frac{1}{3}$ yard long by $4 \frac{1}{2}$ yard wide is 6 sq. yds. (square yards) or 6 yds$^2$.

Solve the following problems.

1) Find the area of a plot of land that is $\frac{3}{5}$ of a mile wide by $\frac{1}{6}$ of a mile long.

2) How many square meters of sod do I need for a section of yard that is $9 \frac{1}{3}$ meters long by $6 \frac{3}{4}$ meters wide?
You will encounter word problems in this course (throughout homework sheets, projects, quizzes and tests) as well as beyond this course. You need to analyze these word problems and decide how you are going to approach them and which method(s) you will use to solve them. The different kinds of word problems presented in this chapter are:

1) FRACTION OF A WHOLE WORD PROBLEMS

Choose the Method that works best for you:

Method 1: Fraction Picture Method
Method 2: Translation Method
Method 3: Proportion Method

2) CUTTING OR SEPARATING WORD PROBLEMS

3) FRACTION RATE PROBLEMS

Choose the Method that works best for you:

Method 1: Translation Method
Method 2: Proportion Method

4) AREA PROBLEMS WITH FRACTIONS

a) Use the formula for Area of a Rectangle (Area = Length x Width; A = L x W).

b) Recall the rules for multiplying with fractions and mixed numbers.

NOTE: You may need a combination of two or more of the above methods to solve certain word problems.

NOTE: It is best to be familiar with all of the methods listed above. You may prefer to use one over another, but sometimes the method you prefer is not the best one to use with a particular problem. Please practice until you are comfortable with all of the problem solving techniques presented in this chapter to best equip yourself to succeed in this course and beyond.
Solve the following problems:

1) If you can hike $3 \frac{1}{3}$ miles in 1 hour, at the same rate, how far can you go in 9 hours?

2) What number is $\frac{3}{4}$ of 60?

3) If $\frac{3}{4}$ of a number is 60, find the number.
4) If two-thirds of a trip is 180 miles, how long is the entire trip?

5) A room is $12 \frac{1}{3}$ feet long and 14 feet wide. If flooring costs $19 \frac{1}{2}$ per sq. ft., installed, how much will it cost to put new flooring in the room?

6) A container is $\frac{3}{8}$ full and has 24 quarts of liquid in it. How much will it hold when it is full? (What is the capacity of the container?)
7) A container holds 24 quarts of liquid when it is full. (The capacity of the container is 24 quarts.) How much liquid will it hold when it is \( \frac{3}{8} \) full?

8) If a seamstress requires \( 2 \frac{3}{5} \) yards of material to make one shirt, how many shirts can be made from 65 yards of material?

9) If a seamstress requires \( 3 \frac{1}{2} \) yards of material to make one shirt, how many shirts can be made from 65 yards of material?
10) A tank holds 150 gallons when it is full. After \(\frac{4}{5}\) of the tank is drained out, how much water is left in the tank?

11) If a rectangular plot in a community garden measures \(\frac{2}{3}\) yd wide and \(\frac{7}{9}\) yd long, how much will this piece of land cost if land sells for $540 per sq. yd.?

12) Suppose you’ve agreed to do a particular job. After working 150 hours, you’ve completed \(\frac{3}{8}\) of the job. What are the two “natural” questions that can be asked here? Write them, and then answer them.
For even **more practice**, solve the following:

13) If one bag of ice weighs \(7 \frac{3}{5}\) lbs, how much would 11 of these bags weigh?

14) If a picture measures \(\frac{3}{8}\) yd. wide and \(\frac{2}{3}\) yd. long, what will you need to spend on a piece of glass to protect this picture if glass costs $12 per sq. yd?

15) If you worked \(18 \frac{3}{4}\) hours and received $206 \(\frac{1}{4}\), what was your hourly rate of pay?
16) If a manufacturer requires $3\frac{3}{8}$ yards of material to make one suit, how many suits can be made from $91\frac{1}{8}$ yards of material?

17) If an entire shipment (consisting of 9 boxes) weighs 360 pounds, how much would $\frac{5}{9}$ of the shipment weigh?? (How much would only 5 boxes weigh?)

18) How many $5\frac{1}{4}$ lb. bags of apples can be obtained from 42 lbs. of apples?
19) If you can ride your bike at the rate of $8 \frac{1}{6}$ miles per hour, how far can you go in 3 hours?

20) Suppose after driving 320 miles you have completed $\frac{5}{8}$ of a trip. How long is the entire trip? How many more miles must you travel in order to complete the trip?”
Before you do your homework please review the terms in this chapter and the method(s) for working with fractions and solving word problems. If you are not familiar with any or all of these methods: Refer to your notes OR see a tutor OR go to Open Time in lab.

Use any appropriate method(s) that will help you arrive at reasonable solutions for the following word problems. Remember to show all work & label all answers.

<p>| | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>1) ( \frac{2}{5} ) of what number is 40?</td>
<td>2) ( \frac{2}{5} ) of 40 is what number?</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3) If one inch represents 15 miles on a map, what does ( \frac{3}{5} ) of an inch (( \frac{3}{5} ) inches) represent?</td>
<td>4) NCC has 20,000 students and ( \frac{4}{5} ) of the students receive some sort of financial aid. How many students do not receive financial aid?</td>
</tr>
</tbody>
</table>
5) How much will it cost to purchase land for a park that measures $\frac{3}{4}$ mile by $\frac{3}{8}$ mile if the land costs $320,000 per square mile?

6) How much will it cost to make a cork board that is $\frac{2}{3}$ yd. by $\frac{3}{5}$ yd. if cork costs $25 per sq. yd.?

7) A wooden dowel that is $\frac{3}{4}$ of a yard long is to be cut into 12 equal length pieces. How long is each piece?

8) A wooden dowel is 12 inches long and is to be cut into pieces that are each $\frac{3}{4}$ inch long. How many pieces can I cut?
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
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<tbody>
<tr>
<td>9) If the capacity of a gas tank is 12 gallons, how many gallons of gas are in the tank if it is ( \frac{3}{4} ) full?</td>
<td>10) When a gas tank is ( \frac{3}{4} ) full it has 12 gallons in it. What is the capacity of the gas tank?</td>
</tr>
<tr>
<td>10) When a gas tank is ( \frac{3}{4} ) full it has 12 gallons in it. What is the capacity of the gas tank?</td>
<td></td>
</tr>
<tr>
<td>11) A sweet potato pie has 3000 calories in the whole pie. If each portion (serving) of the pie is ( \frac{1}{8} ) of the pie, how many calories are in each slice (one serving)?</td>
<td>12) Overtime pay is ( 1\frac{1}{2} ) times the regular rate of pay. If the regular rate of pay is $12 per hour, what is the overtime rate of pay?</td>
</tr>
</tbody>
</table>
13) There are 84 students in all of Prof. A’s classes combined and \( \frac{4}{7} \) of them have passed the MAT 001 exit exam. How many of Prof. A’s students passed the MAT 001 exit exam?

14) 84 of Prof. A’s students passed the MAT 001 exit exam and this is \( \frac{4}{7} \) of Prof. A’s students. How many students does Prof. A. have all together?

15) A piece of wood that is \( 3 \frac{1}{3} \) yards long is to be cut it into 5 equal length pieces. How long is each piece?

16) The high-flying swing ride at Adventure Land makes \( 4 \frac{2}{7} \) revolutions per minute. If the ride makes 10 revolutions, how many minutes did the ride last?
17) A piece of wood that is 15 yards long is to be cut into pieces that are each \(3 \frac{1}{3}\) yards long. How many pieces of this size can be cut from this length of wood?

18) A gas tank holds \(14\frac{1}{2}\) gallons of gas when it is filled to capacity. If gas costs \$3\frac{1}{2}\) per gallon, how much will it cost to fill this gas tank if the driver runs out of gas?

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
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<tbody>
<tr>
<td>17)</td>
<td>A piece of wood that is 15 yards long is to be cut into pieces that are each (3 \frac{1}{3}) yards long. How many pieces of this size can be cut from this length of wood?</td>
</tr>
<tr>
<td>18)</td>
<td>A gas tank holds (14\frac{1}{2}) gallons of gas when it is filled to capacity. If gas costs $3\frac{1}{2}) per gallon, how much will it cost to fill this gas tank if the driver runs out of gas?</td>
</tr>
</tbody>
</table>

19) At the new food pantry on campus we have 24 pounds of peanuts and they need to be put into bags that hold \(1\frac{1}{2}\) pounds each. How many bags can we fill?

20) Godiva chocolate costs \$32\frac{1}{2}\) per pound. How much will it cost if I buy my chocoholic mother \(2\frac{2}{5}\) pounds for her birthday?
Challenge Problems:

21a) If you can ride $7\frac{1}{2}$ miles per hour on your bike and you live 4 miles from school, how long will it take to ride your bike to school? (Express the answer as a fraction of an hour.)

b) How long is this in minutes?

22) A room is 10 yards wide and 12 yards long. If carpet costs $8 \frac{1}{2}$ per sq. yd., how much will I have to pay to carpet only $\frac{4}{5}$ of this room?
**Area**

*Definition:* Area measures the size of a flat surface using square units. The formula for the area of a rectangle is Area = Length x Width, meaning:

\[ \text{Total # of squares} = \text{# of squares per row} \times \text{# of rows} \]

*Example:* 4 squares/row x 3 rows = 12 squares

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<thead>
<tr>
<th>1</th>
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</table>

This rectangle is 12 square units

**Cube**

*Definition:* A cube of a number is the number times itself three times.

*Example:* The cube of 4 is \(4^3 = 4 \times 4 \times 4 = 64\)

**Denominator**

*Definition:* A denominator is the bottom number in a fraction. It is the total number of equal size portions you break one whole thing into.

*Example:* If the denominator is 4, break the whole thing into 4 equal size portions.

\[
\begin{array}{cccc}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\end{array}
\]

**Divisor**

*Definition:* A divisor is the number that is being repeatedly subtracted from an original number to determine how many complete groups are contained in the original number.

*Example:* In the division sentence:

\[12 \div 3 = 4\]

3 is the divisor and there are 4 full groups of 3 contained in the number 12.

**Exponents (Powers)**

*Definition:* An exponent (power) is the number in an exponential expression that tells you how many times to repeatedly multiply the base by itself.

*Example:* In the exponential expression \(5^4\), the exponent is 4, so the factor 5 is multiplied by itself 4 times.

\[5 \times 5 \times 5 \times 5 = 625\]

**Factor**

*Definition:* Factors are the numbers being multiplied.

*Example:* In the multiplication sentence:

\[12 \times 3 = 36\]

12 and 3 are two factors of 36.

**Fraction**

*Definition:* A fraction is any number expressed as a numerator over a denominator.

*Example:* \(\frac{N}{D}\) \(N = \) Any Whole Number \(D = \) Any Natural Number
Identity Property of Multiplication

**Definition:** The identity property of multiplication states that multiplying a number by 1 gives the same number as the product.

**Example:** \(18 \times 1 = 18\), (18 kept its “identity”).

Improper Fraction

**Definition:** An improper fraction is a fraction where the numerator is greater than or equal to the denominator.

**Example:** \(\frac{5}{3}, \frac{13}{13}, \frac{12}{8}\) are all examples of improper fractions.

Mixed Number

**Definition:** A mixed number is a whole number combined with a proper fraction.

**Example:** \(3 \frac{4}{9}, 7 \frac{1}{2}, 13 \frac{12}{15}\) are all mixed numbers.

Numerator

**Definition:** A numerator is the top number in a fraction which represents the number of equal size portions you want.

**Example:** If the numerator is 3 then you want three portions. (In other words, shade three portions.)

| \(\frac{1}{4}\) | \(\frac{1}{4}\) | \(\frac{1}{4}\) | \(\frac{1}{4}\) |

Perfect Square

**Definition:** A perfect square is a result of multiplying any whole number by itself.

**Example:** \(0^2 = 0\), \(1^2 = 1\), \(2^2 = 4\), \(3^2 = 9\), \(4^2 = 16\)...

The perfect squares are 0, 1, 4, 9, 16...

Prime Factorization

**Definition:** A prime factorization is the writing of a composite number as a product of its prime factors.

**Example:** 12 can be written as the product of the prime factors 2 and 3.

\(12 = 2 \times 2 \times 3\) (Think of “Factor Trees”).

Prime Factorization with Exponents

**Definition:** A prime factorization that uses exponents for repeated prime factors instead of writing them separately.

**Example:** \(12 = 2^2 \times 3\)

Prime Number

**Definition:** A prime number has exactly two different factors, 1 and itself.

**Example:** The factors of 2 are 1 and 2 so 2 is a prime number.

The first ten prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

Product

**Definition:** A product is the answer to a multiplication problem.

**Example:** In the multiplication sentence \(4 \times 5 = 20\), the product is 20.

Proper Fraction

**Definition:** A proper fraction is a fraction where the numerator is smaller than the denominator.

**Definition:** A proper fraction represents a piece or a portion of a whole thing.

**Example:** \(\frac{2}{3}, \frac{5}{11}, \frac{4}{8}\) are examples of proper fractions.

Quotient

**Definition:** A quotient is the answer to a division problem.

**Example:** In the division sentence \(20 \div 5 = 4\), the quotient is 4.

Remainder

**Definition:** A remainder is whatever is left after you have subtracted the divisor as many times as possible from a given quantity.

**Example:** In the division sentence, \(22 \div 5 = 4r2\), because \(22 - 5 - 5 - 5 - 5 = 2\), the quotient is 4 and the remainder is 2.

Square

**Definition:** A square of a number is the number squared, that is, the number times itself.

**Example:** The Square of 36 = \(36^2 = 1296\)

Square Root

**Definition:** The square root of a given number asks you to determine what number must be multiplied by itself to get the number under the radical sign

**Example:** The Square Root of 36 = \(\sqrt{36} = 6\)
Converting Improper Fractions to Mixed Numbers: Convert an Improper Fraction to a Mixed Number by Dividing the Numerator by the Denominator. The Quotient is the Whole Number and the Remainder is the Numerator of the Proper Fraction. Keep the Denominator, it does not change.
See page 209.

Converting Mixed Numbers to Improper Fractions: To convert a mixed number to an improper fraction: Multiply the Denominator by the Whole Number then add the Numerator of the Proper Fraction to create the new Numerator for the Improper Fraction. Keep the Denominator, it does not change.
See page 208.

Cross Multiplying: To cross multiply we multiply the numerator of one fraction by the denominator of the other fraction only across an equal sign.
See pages 216, 217, 253, 382.

Cutting/Separating into Equal Size Pieces: When cutting or separating something into equal size pieces or portions, it is being “evenly divided.” Start with the thing that is being cut or separated followed by the division sign followed by the divisor which is either the number of equal portions or the size of each of portion.
See page 124-126, 184, 186, 275, 276.

Fraction Division: In fraction division you must change the operation to multiplication by the reciprocal of the divisor. Memory Trick: “Keep Times Flip” i.e. Keep the first fraction the same, change ÷ to x, and flip the second fraction.
See page 239.

Fraction Multiplication: A product of fractions is the result of the product of the numerators over the product of the denominators. All fraction answers must be written in lowest terms. Because of this, it’s easier to reduce first with Algebra before the multiplication:
Remember: Stretch then Factor, Cancel, and Multiply.
See page 228.

Reciprocal/Finding a Reciprocal: To find the reciprocal of any number (except 0) put the number in fraction form and then switch the numerator and denominator (or “flip it”).
See page 239.

Reducing Fractions to Lowest Terms: A fraction is in lowest terms when the numerator and the denominator have no prime factors in common. To reduce a fraction algebraically, remember to Factor, Cancel, Multiply.
See pages 219 - 220.

Renaming Equivalent Fractions: Equivalent fractions are two fractions that appear different but have the same numerical value. To create an equivalent fraction from an existing fraction multiply it by 1 written as $\frac{2}{2} \cdot \frac{3}{3} \cdot \frac{4}{4}$, etc. When you multiply the existing fraction by 1 (in fractional form) you multiply the numerator and denominator by the same number and build a new equivalent fraction.
See page 215 - 216.

Testing for Equal Fractions: To see if two fractions are equal or if two ratios are proportional, cross multiply. If the cross products are equal then the fractions are equal or the two ratios are proportional or check to see if the quotients are identical when doing top ÷ bottom.
See page 217.
For questions 1 – 3, indicate what fractional part is shaded:

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<thead>
<tr>
<th>1)</th>
<th>2)</th>
<th>3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
</tbody>
</table>

For questions 4 – 6, draw a picture to represent the given fraction:

<table>
<thead>
<tr>
<th>4)</th>
<th>5)</th>
<th>6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4</td>
<td>3/5</td>
<td>5/2</td>
</tr>
</tbody>
</table>

7) If you eat 3 slices of a 4-slice personal pan pizza, what fraction of a pie remains?

8) If you buy 2 personal pan pizzas (4 slices each) and eat 5 slices, what fraction of a pie remains?

For questions 9 – 12, change the improper fractions to mixed numbers and change the mixed numbers to improper fractions:

<table>
<thead>
<tr>
<th>9)</th>
<th>10)</th>
<th>11)</th>
<th>12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/5</td>
<td>122/5</td>
<td>3/7</td>
<td>32 3/11</td>
</tr>
</tbody>
</table>

Mathematical Foundations by Debra J. Grodenchik, Ph.D., Jennifer W. Kohut, et al at Nassau Community College is licensed under [CC-BY-SA 4.0](https://creativecommons.org/licenses/by-sa/4.0/)
For questions 1 – 4, solve for n:

1) \( \frac{n}{4} = \frac{6}{12} \)
2) \( \frac{12}{n} = \frac{4}{5} \)
3) \( \frac{6}{15} = \frac{n}{40} \)
4) \( \frac{65}{40} = \frac{13}{n} \)

For questions 5 – 8, test for equality:

5) \( \frac{2}{3} = \frac{5}{6} \) ?
6) \( \frac{5}{6} = \frac{15}{16} \) ?
7) \( \frac{5}{6} = \frac{15}{18} \) ?
8) \( \frac{25}{16} = \frac{5}{6} \) ?

For questions 9 – 11, reduce the following fractions to lowest terms (simplify completely):

9) \( \frac{16}{300} \)
10) \( \frac{45}{75} \)
11) \( \frac{1250}{500} \)
1) Show a picture \( \frac{1}{3} \) of 6

2) Show a picture \( \frac{1}{3} \) of \( \frac{1}{2} \)

For questions 3 – 18, evaluate the following:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3)</td>
<td>[ \frac{7}{11} \times \frac{3}{2} ]</td>
<td>4)</td>
</tr>
<tr>
<td>5)</td>
<td>[ \frac{3}{13} \times \frac{13}{19} ]</td>
<td></td>
</tr>
<tr>
<td>6)</td>
<td>[ 1 \frac{1}{3} \times 2 \frac{1}{2} ]</td>
<td></td>
</tr>
<tr>
<td>7)</td>
<td>[ 2 \frac{1}{2} \times 2 \frac{1}{3} ]</td>
<td></td>
</tr>
<tr>
<td>8)</td>
<td>[ \frac{3}{2} \times \frac{5}{18} \times \frac{6}{7} ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
| 9) | \[
\frac{27}{21} \times \frac{13}{15} \times \frac{28}{39}
\] | 10) | \[
\frac{3}{3} \times \frac{1}{4} \times \frac{12}{25}
\] |
| 11) | \[
\left(\frac{9}{25}\right)^2
\] | 12) | \[
\sqrt{1\frac{11}{25}}
\] |
| 13) | \[
\sqrt{\frac{16}{49}}
\] | 14) | \[
\left(2\frac{1}{3}\right)^2
\] |
| 15) | Find the square of \(\frac{9}{16}\) | 16) | Find the square of \(1\frac{4}{9}\) |
| 17) | Find the square root of \(\frac{9}{16}\) | 18) | Find the square root of \(1\frac{9}{16}\) |
1) What is the reciprocal of 5?  
2) What is the reciprocal of $\frac{2}{3}$? 

3) What is the reciprocal of $1\frac{2}{3}$?  
4) What is the reciprocal of $\frac{1}{18}$? 

For questions 5 – 10, find the following quotients and simplify:

<table>
<thead>
<tr>
<th>5) $\frac{2}{3} \div 3$</th>
<th>6) $3 \div \frac{2}{3}$</th>
<th>7) $\frac{14}{3} \div \frac{7}{9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8) $\frac{7}{9} \div \frac{14}{3}$</td>
<td>9) $2\frac{1}{4} \div 4$</td>
<td>10) $4 \div 2\frac{1}{4}$</td>
</tr>
</tbody>
</table>
1) What is $\frac{1}{3}$ of 6?

2) What is $\frac{1}{3}$ of $\frac{1}{2}$?

3) $\frac{3}{4}$ of what number is 24?

4) What is $\frac{3}{4}$ of 24?

5) $\frac{3}{4}$ of what number is 12?

6) $\frac{1}{3}$ of what number is $\frac{1}{9}$?

7) What fractional part of $\frac{3}{4}$ is $\frac{1}{2}$?
1) How much will it cost to purchase land for a park that measures 1/2 mile by 3/4 mile if land costs $400,000 per square mile?

2) How much will it cost me to make a cork board that is 3/4 yd. by 2/3 yd. if cork costs $30 per sq. yd.?

3) If one inch represents 20 miles on a map, what does 2/5 inches (2/5 of an inch) represent?

4) 2/3 of my students have a lab grade of 65 or higher on their mid-semester lab report. If I have 66 students, how many of my students have a lab grade of at least 65?

5) I have a wooden dowel that is 2/3 of a yard long. I must cut it into 6 equal length pieces. How long is each piece (in yards)? How long is that in inches?

6) I have a wooden dowel that is 6 inches long and I need pieces that are each 2/3 inch long. How many pieces can I cut?
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>7) The capacity of my gas tank is 15 gallons. How many gallons of gas are in the tank if it is 3/5 full?</td>
<td></td>
</tr>
<tr>
<td>8) If my gas tank is 3/5 full when it has 15 gallons in it, what is the capacity of my gas tank?</td>
<td></td>
</tr>
<tr>
<td>9) 3/5 of my 60 students passed BEP 092 with at least a 75 average last semester. How many of my students passed with a 75 average or higher?</td>
<td></td>
</tr>
<tr>
<td>10) 60 of my students passed BEP 092 with at least a 75 average last semester. If this is 3/5 of my students, how many students were enrolled in my BEP 092 classes last semester?</td>
<td></td>
</tr>
</tbody>
</table>
Challenge problems:

11) I have a piece of wood that is $15 \frac{1}{2}$ yards long and I need to cut pieces that are each $1\frac{1}{2}$ yards long. How many pieces can be cut?

BONUS: if there is a fraction of a piece left over, how long is this extra piece of wood?

12) A room is 15 yards wide and 12 yards long. If carpet costs $7 \frac{1}{2}$ per sq. yd., how much will I have to pay to carpet only $\frac{2}{3}$ of this room?
3.1 – extra practice answers:
1) 5/6
2) 4/6 which reduces to 2/3
3) 5/3
4) 
5) 
6) 
7) 1/4
8) 3/4
9) \(\frac{\text{2}}{5}\)
10) \(\frac{\text{24}}{5}\)
11) 17/7
12) 355/11

3.2 extra practice answers:
1) n = 2
2) n = 15
3) n = 16
4) n = 8
5) no
6) no
7) yes
8) no
9) 4/75
10) 3/5
11) 5/2

3.3 extra practice answers:
1) 
2) 
3) 21/22
4) 1/3
5) 3/19
6) \(\frac{3}{5}\)
7) \(\frac{5}{6}\)
8) 5/14
9) 4/5
10) 2
11) 81/625
12) \(\frac{1}{5}\)
13) 4/7
14) \(\frac{5}{9}\)
15) 81/256
16) \(\frac{7}{81}\)
17) 3/4
18) \(\frac{1}{4}\)

3.4 extra practice answers:
1) 1/5
2) \(\frac{\text{3}}{2} = \frac{\text{1}}{\text{2}}\)
3) 3/5
4) 18
5) 2/9
6) \(\frac{9}{2} = \frac{4}{\text{2}}\)
7) 6
8) 1/6
9) 9/16
10) \(\frac{1}{\text{7}}\)
Extra Practice/chapter review  ANSWERS

3.5 extra practice answers:

1) 2  2) $1/6  3) 32  4) 18
5) 16  6) 1/3  7) 2/3

3.6 extra practice answers:

1) $150,000  2) $15  3) 8 miles
4) 44 students  5) 1/9 yard = 4 inches  6) 9 pieces
7) 9 gallons  8) 25 gallons  9) 36 students
10) 100 students

Challenge Problems:

11) 10 pieces with an extra 1/2 yard  12) $900
CHAPTER 4

PROBLEM SOLVING

WITH

DECIMALS
Chapter 4.1 DECIMAL INTRODUCTION

**Decimals** and **fractions** are different forms of the same number.

Every terminating (ex. 0.5) and repeating decimal (ex. 0.\(\overline{3}\)) has an equivalent fraction.

Decimals are used more for money, science, and sports:

\[
\frac{1}{2} \text{ of a dollar is the same as $0.50$
\]

Gravity is measured as 9.8 meters per second squared = 9\(\frac{4}{5}\) meters per second squared

Simone Manuel’s 2016 gold medal winning, Olympic record time in the 100 meter women’s freestyle was 52.7 seconds = 52\(\frac{7}{10}\) seconds

The **whole number part** gives the number of whole things you have.

The **decimal portion** is only a piece of one whole thing.

**Whole Number Part , Decimal Portion**

**The decimal point separates** the whole number part from the decimal portion.

Definition: **Decimal-fraction** – any **fraction** whose denominator is a power of 10 (that is, the denominator = 10, 100, 1000, etc.)

\[
\begin{align*}
\frac{1}{10} &= \frac{1}{10^1} = 0.1 \\
\frac{1}{100} &= \frac{1}{10^2} = 0.01 \\
\frac{1}{1000} &= \frac{1}{10^3} = 0.001 \\
\frac{1}{10,000} &= \frac{1}{10^4} = 0.0001
\end{align*}
\]

Notice that the number of decimal places in the decimal notation is equal to the number of zeros in the denominator of the first fraction and equal to the exponent of the number 10 in the denominator of the fraction on the right.

**NOTE:** The word name for a **decimal** is the same as the word name for its **decimal-fraction**.

**Ex. A:** Write the word name for \(\frac{3}{10}\) and for 0.3.

\(\frac{3}{10}\) can be read as “three over ten” but it can also be read as “three tenths.” Three tenths is also its formal decimal word name.

**Answer:** The word name for \(\frac{3}{10}\) and for 0.3 is three tenths.
WRITING DECIMAL WORD NAMES

To write decimal word names, you first must know your **decimal place values**:

```
6 2 3 5 . 4 6 7 5 9 1 3

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
<th>Ten-thousandths</th>
<th>Hundred-thousandths</th>
<th>Ten-millionths</th>
</tr>
</thead>
</table>
```

**ZERO TRICK**: The number of decimal places (to the right of the decimal point) EQUALS the number of **zeros after the 1** in a whole number power of 10.

**Naming Decimal Place Values**

**Method 1**: Use the zero trick while writing zeros under the decimal places.

**Ex. A**: What is the place value name for the digit 7 in 6235.4675913?

To use the zero trick you must first locate the ones place. It is just to the left of the decimal point.

```
6 2 3 5 . 4 6 7 5 9 1 3
```

_Step 1_: Write a 1 under the digit in the ones place. In this example it is the digit 5.

_Step 2_: Add a zero to the right of the decimal point for each digit up to and including the 7.

_Step 3_: Read the number created under 1234.5678912, and add the ending **ths** → thousandths.

**Answer**: The seven is in the thousandths (1000ths) place in 6235.4675913.

**Method 2**: Use the zero trick by counting decimal places and writing a correct power of 10.

**Ex. B**: What is the place value name for the 9 in 6235.4675913?

According to the **zero trick**, the 9 is in the 5th decimal place:

Five decimal places (5 places to the right of the decimal point) tells us there are five zeros after the 1 in the whole number power of 10 for the answer. Write a 1 followed by 5 zeros & the ending **ths**. This gives us 100,000ths

**Answer**: The 9 is in the hundred thousandths (100,000ths) place in 6235.4675913.
HOW TO WRITE DECIMAL WORD NAMES

Method 1:

Step 1: Change the decimal to an **unreduced** decimal fraction (use the zero trick on page 313).
Step 2: Add the suffix “ths” to the denominator.
Step 3: Write the word name for the decimal fraction.

**Ex. A:** Write the word name for the number, 12.023.

```
12 . 0 2 3 = \( \frac{23}{1000} \) ths
```

Now write the word name for \( \frac{23}{1000} \) ths → twelve and twenty-three thousandths.

**Answer:** The word name for 12.023 is **twelve and twenty-three thousandths**.

**NOTE:** Remember, any fraction is made from two whole numbers, one for the numerator and one for the denominator. When you write 023 in the numerator, drop any “leading” zeros. These are zeros that appear between the decimal point and the nonzero digits: 023 = 23

**Ex. B:** Write a word name for 0.106.

The decimal 0.106 could be written as .106: The whole number in 0.106 is 0 and the reason for writing the 0 first is to make sure you see the decimal point. Because the whole number part is zero, this decimal number is equivalent to a proper fraction.

```
0 . 1 0 6 = \( \frac{106}{1000} \) ths
```

**Answer:** The word name for 0.106 is **one hundred six thousandths**.
Method 2:

Ex. A: Write the word name for the number, 12.023.

Step 1: Write the word name for the whole number part (to the left of the decimal point).
Step 2: Write the name for the decimal point “and.”
Step 3: Write the word name for the decimal side (to the right of the decimal point) as if it were a whole number.
Step 4: Write the decimal place value name (the “ths” name) for the last digit on the right, using the zero trick here.

12 . 023

Write the name for the whole number part (to the left of the decimal point).
Write the name for the decimal point “and.”
Write the word name for the decimal side (to the right of the decimal point) as if it were a whole number. Ignore leading zeros.
Write the decimal place value name (the “ths” name) for the last digit on the right, using the zero trick here: 3 decimal places → 3 zeros after the 1 in the decimal place value name. 1000ths → thousandths

twelve and twenty-three thousandths

Answer: The word name for 12.023 is twelve and twenty-three thousandths.

Ex. B: Write a word name for 0.106

Step 1: Write the name for the whole number part (to the left of the decimal point).
Step 2: Write the name for the decimal point “and.”

NOTE: The whole number in 0.106 is 0 and the reason for writing the 0 first is to make sure you see the decimal point. The zero “adds” nothing to the number. So, you can skip Steps 1 and 2.

Start with Step 3:

Step 3: Write the word name for the decimal side (to the right of the decimal point) as if it were a whole number.
Step 4: Write the decimal place value name (the “ths” name) for the last digit on the right, using the zero trick here.
Write the word name for the **decimal side** (to the right of the decimal point) *as if it were a whole number.*

Write the decimal place value ("ths") name for the **last digit on the right** using the zero trick:
3 decimal places → 3 zeros after the 1 in the decimal place value name: 1000ths → thousandths

one hundred six thousandths

**NOTE:** When the whole number part is zero we skip to the decimal side.
(It sounds funny to start with "zero and” when writing a decimal word name.)

**Answer:** The word name for 0.106 is **one hundred six thousandths.**

Write a decimal word name for the following:

1) 0.35
2) 0.0006

3) 102.201
4) 45.0304

5) 12.00201
6) 2,012.2003

**NOTE:** The first decimal place is “tenths.” The name for the decimal point is “and.”
WRITING STANDARD DECIMAL NOTATION FROM A DECIMAL WORD NAME

To write decimal notation from a word name there are two methods.

**Method 1:**

**Step 1:**
Look for the word “and” (the decimal point) and circle it.
Remember, it separates the whole number from the decimal part.

NOTE: If there is no word “and” it means the whole number is 0 and you should skip to **Step 3**.

**Step 2:**
Circle the whole number word name that precedes the word “and”.
Write the number in standard notation to the left of the decimal point

**Step 3:**
Circle the decimal place value name. It is the word name that has “ths” at the end.
Write the place value number; 10ths, 100ths, 1000ths, 10,000ths….
The number of zeros is the number of decimal places in the decimal.
Write a decimal point and as many blanks to the right of the decimal point as there are zeros in the place value name. For example, 10ths gets one blank, 100ths gets two blanks, 1000ths gets three blanks, etc.

**Step 4:**
Circle what’s left. (It’s a word name between the “and” and the “ths” name.) Write it in standard notation: 13 is a 2-digit number, so put it in the last 2 blanks on the right. Fill in any extra blanks with zeros.

**Ex. A:** Write standard decimal notation for forty-one and thirteen thousandths.

**Answer:** Forty-one and thirteen thousandths is 41.013 in decimal notation.
**Ex. B:** Write decimal notation for thirty-five ten thousandths.

Since there is a decimal place value ("ths") name at the end of the word name, then this number has a decimal portion. If there is no "and," then the whole number part is zero. Write a zero followed by a decimal point and skip to **Step 3.**

**Step 3:** Circle the place value ("ths") name. 10,000ths has 4 zeros = 4 decimal places, so write 4 blanks on the decimal side.

**Step 4:** Circle what’s left. (It’s the word name before the “ths” name.) Write it in standard notation: 35 is a 2-digit number, so put it in the last 2 blanks on the right. Fill in any extra blanks with zeros.

Answer: Thirty-five ten thousandths is 0.0035 in decimal notation.

**NOTE:** If you have “ten” or “hundred” just to the left of the place value name ("ths") name, that “ten” or “hundred” is part of that place value name ("ths" name).

**NOTE:** You never end a decimal number with zeros unless you have a specific reason to do so.

**Method 2:**
This method uses the fact that word names for fractions/mixed numbers and decimals are the same. This method requires the use of a calculator.

**Ex. A:** Write decimal notation for forty-one and thirteen thousandths.

Write the **mixed number** for forty-one and thirteen thousandths.

Enter this into your scientific calculator.

\[
\text{forty-one and thirteen thousandths} = 41 \frac{13}{1000} = 41 \text{ and } \frac{13}{1000} = 41 + 13 \div 1000 = 41.013
\]

Answer: Forty-one and thirteen thousandths is 41.013 in decimal notation.

**NOTE:** You must use order of operations if you don’t have a scientific calculator.
Ex B: Write decimal notation for three and seven hundred five hundred thousandths.

Write the mixed number for three and seven hundred five hundred thousandths

\[
3 \frac{705}{100,000} = 3 + \frac{705}{100,000} = 3 + 705 \div 100,000 = 3.00705
\]

Answer: Three and seven hundred five hundred thousandths is 3.00705 in decimal notation.

NOTE: If you have “ten” or “hundred” just to the left of the place value name (“ths” name), that “ten” or “hundred” is part of that place value name (“ths” name).

NOTE: You never end a decimal number with zeros unless you have a specific reason to do so.

Problems - Write the following in decimal notation using both methods.

1) Forty-five ten thousandths

2) Seventeen and one hundred thirteen hundred thousandths

3) Forty-three thousand two and two thousand, three hundred four hundred millionths

4) Nineteen thousand, three hundred four millionths

Enter this into your scientific calculator.
Homework 4.1  Name:

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Decimal Fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Word Names</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Standard Notation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Place Value</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For questions 6 – 8, give the name of the place value for the digit 6 in the following numbers:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6)</td>
<td>307.16</td>
<td>7)</td>
</tr>
</tbody>
</table>

For questions 9 – 14, write a **word name** for each of the following numbers:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9)</td>
<td>14.07</td>
<td>10)</td>
</tr>
<tr>
<td>11)</td>
<td>0.140007</td>
<td>12)</td>
</tr>
<tr>
<td>13)</td>
<td>1231.2</td>
<td>14)</td>
</tr>
</tbody>
</table>
For questions 15 – 17, in the number **1357.2468** give the digit that represents the following place values:

<table>
<thead>
<tr>
<th></th>
<th>Hundreds</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>15)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 18 – 23, write the following word names in standard notation:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18)</td>
<td>Fifteen hundredths</td>
<td>19) Two thousand five and one hundred one thousandths</td>
</tr>
<tr>
<td>19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20)</td>
<td>Eighty seven and nine hundred fifty three hundred-thousandths</td>
<td>21) Fifteen and eleven ten-thousandths</td>
</tr>
<tr>
<td>21)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22)</td>
<td>Eight and one hundred eight ten-thousandths</td>
<td>23) Sixteen thousand eight millionths</td>
</tr>
<tr>
<td>23)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 4.2

CONVERTING FRACTIONS TO DECIMALS 
AND 
DECIMALS TO FRACTIONS

Converting Fractions to Decimals:

Recall that fractions are a form of division. One of the forms of division notation is fraction notation. $12 \div 3 = 4$ can be written as $\frac{12}{3} = 4$. Now, that we have covered fractions in Chapter 3 we don’t assume the “larger” number is the numerator.

A fraction is a notation for division: $\frac{3}{12} = 3 \div 12 = 0.25$

The easiest way to write fractions as decimals is to use the division rule:

$$\frac{\text{Numerator}}{\text{Denominator}} = \text{Numerator ÷ denominator}$$

Ex. A: Write $\frac{3}{5}$ as a decimal.

Use the division rule: $\frac{3}{5} = 3 ÷ 5 = 0.6$

Answer: $\frac{3}{5}$ is 0.6 as a decimal.

Ex. B: Write $\frac{4}{9}$ as a decimal.

Use the division rule: $\frac{4}{9} = 4 ÷ 9 = 0.444444444……. $ This is a repeating decimal. The digit 4 will repeat for all eternity. Rather than write this we indicate the repeating digit(s) by writing a bar over the digit (or digits, if they are different) that repeat. So, $\frac{4}{9} = 0.444444444……. = 0.\overline{4}$

Answer: $\frac{4}{9}$ is $0.\overline{4}$ as a decimal.

What if you want to convert a mixed number to a decimal? There are a couple of options. Remember a mixed number consists of a whole number (WN) and a fraction ($\frac{n}{d}$).

Enter this into your scientific calculator.

$$\text{WN} \frac{n}{d} = \text{WN} + (n ÷ d) = \text{Decimal}$$

NOTE: This works for a scientific calculator. If you have a basic four function calculator you must convert the mixed number to an improper fraction first, then divide: numerator ÷ denominator.
**Ex. C:** Write $5 \frac{3}{4}$ in decimal notation.

Using the formula:

\[
WN \frac{n}{d} = WN + (n \div d) = \text{Decimal}
\]

\[
5 \frac{3}{4} = 5 + (3 \div 4) = 5.75
\]

OR

\[
5 \frac{3}{4} = \frac{23}{4} = 23 \div 4 = 5.75
\]

**Answer:** $5 \frac{3}{4}$ is 5.75 in decimal notation.

**Ex. D:** Write $24 \frac{5}{6}$ in decimal notation.

Using the formula:

\[
WN \frac{n}{d} = WN + (n \div d) = \text{Decimal}
\]

\[
4 \frac{5}{6} = 4 + (5 \div 6) = 4.83333333… = 4.8\bar{3}
\]

OR

\[
4 \frac{5}{6} = \frac{29}{6} = 29 \div 6 = 4.83333333… = 4.8\bar{3}
\]

**Answer:** $24 \frac{5}{6}$ is 24.8333333 in decimal notation.

Write the following fractions or mixed numbers in decimal notation:

1) \( \frac{1}{2} \) \quad 2) \( \frac{5}{8} \) \quad 3) \( \frac{2}{3} \)

4) \( \frac{3}{9} \) \quad 5) \( \frac{5}{2} \) \quad 6) \( \frac{7}{8} \)

7) \( \frac{4}{9} \) \quad 8) \( \frac{5}{16} \) \quad 9) \( \frac{7}{11} \)
CONVERTING DECIMALS TO FRACTIONS

Converting decimals to fractions is a little more work than converting fractions and mixed numbers to decimals. Remember, the fraction in your answer has to be reduced to lowest terms.

The denominators for decimal fractions are powers of ten.

\[
\begin{align*}
10 &= 10^1 = 2^1 \times 5^1 \\
100 &= 10^2 = 2^2 \times 5^2 \\
1000 &= 10^3 = 2^3 \times 5^3 \\
10,000 &= 10^4 = 2^4 \times 5^4 \\
\end{align*}
\]

etc.

NOTE: Recall that to reduce a fraction to lowest terms you have to divide both the numerator and the denominator by the same number at the same time.

NOTE: The denominators of decimal fractions are all powers of ten. This means their prime factorizations only contain powers of 2 and 5. Therefore, the denominators are only divisible by 2 and 5 (or powers of 2 and 5).

When reducing decimal fractions you only need to see if the numerator is divisible by 2 or 5.

Converting Decimals to Fractions:

**Step 1:** Write the whole number for the decimal as the whole number for the mixed number. (If the whole number is not there or is zero skip this step and go to **Step 2**.)

**Step 2:** Write the decimal portion as the numerator of the fraction (as a whole number).

**Step 3:** Write the denominator as a power of ten: a 1 followed by as many zeros as there are decimal places.

**Step 4:** Reduce the fraction to lowest terms by dividing both the numerator and denominator by 2 or 5. Repeat the process until either the numerator or the denominator is no longer divisible by 2 or 5.
**Ex. A:** Write 3.04 as a fraction or mixed number.

$$\frac{3}{1} \div \frac{4}{2} = \frac{3}{2} \div \frac{2}{2} = \frac{3}{1} \div \frac{1}{25}$$

(answer)

**Step 1:** The Whole Number is the same for both the decimal and the mixed number. Here it is the number 3.

**Step 2:** The decimal portion becomes the numerator. Here the decimal portion is 04 → 4, which is the numerator.

**Step 3:** There are two decimal places so the denominator is a 1 followed by two zeros: 100.

**Step 4:** The numerator is even so the fraction can be reduced by dividing the numerator and the denominator by 2.

```
Answer: 3.04 = 3 \frac{1}{25} as a mixed number.
```

**Ex. B:** Write 0.125 as a fraction.

$0.125 = \frac{1}{8}$ as a fraction.

(answer)

**Step 1:** The whole number is 0 so skip to Step 2.

**Step 2:** The decimal portion becomes the numerator. Here the decimal portion is 125, which is the numerator.

**Step 3:** There are three decimal places so the denominator is a 1 followed by three zeros: 1000.

**Step 4:** The numerator ends in a 5, so the fraction can be reduced by dividing the numerator and the denominator by 5.

```
Answer: 0.125 = \frac{1}{8} as a fraction.
```
Write the following in fraction notation:

1) 0.0006
2) 0.035
3) 0.238
4) 53.625
5) 0.02
6) 0.45
7) 0.402
8) 6.6
For those who don’t like calculators and are good at remembering “tricks” this one’s for you…

If you have a fraction that can EASILY be turned into a decimal-fraction (if you can easily turn the denominator into a 10, 100 or 1000, etc using multiplication) then you can do the following:

Think of one of the two basic facts: \(2 \times 5 = 10\) and \(4 \times 25 = 100\)

**NOTE:** Remember multiplication is commutative: \(5 \times 2 = 10\) and \(25 \times 4 = 100\)

**Ex A:** Write \(\frac{1}{250}\) as a decimal.

<table>
<thead>
<tr>
<th>When multiplying think: 25 x 4 = 100 with an extra zero 250 x 4 = 1000.</th>
<th>Remember to multiply both the numerator and the denominator by 4.</th>
<th>Now you have a decimal-fraction and you are done. Remember, the name for this decimal-fraction is the same as its decimal name!</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{250} \times \frac{4}{4} = \frac{4}{1000} = 0.004)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answer:** \(\frac{1}{250}\) is 0.004 written as a decimal

**Problems:** Write the following in decimal notation (using decimal-fractions):

1) \(\frac{1}{20} = \)

2) \(\frac{4}{5} = \)

3) \(\frac{12}{25} = \)

4) \(\frac{11}{40} = \)

5) \(\frac{7}{2500} = \)

6) \(\frac{11}{5000} = \)
COMMON FRACTION/DECIMAL EQUIVALENTS

Certain tasks in mathematics are made easier when you can recognize common fraction/decimal equivalents.

The following decimal/fraction equivalents are the most common that everyone should know:

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Fraction Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>0.25</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>0.75</td>
<td>$\frac{3}{4}$</td>
</tr>
</tbody>
</table>

Other common decimal/fraction equivalent that are helpful to know:

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Fraction Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$\frac{1}{10}$</td>
</tr>
<tr>
<td>0.2</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>0.4</td>
<td>$\frac{2}{5}$</td>
</tr>
<tr>
<td>0.6</td>
<td>$\frac{3}{5}$</td>
</tr>
<tr>
<td>0.8</td>
<td>$\frac{4}{5}$</td>
</tr>
</tbody>
</table>

The next chart shows repeating decimals:

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Fraction Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>0.6</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>0.1</td>
<td>$\frac{1}{9}$</td>
</tr>
</tbody>
</table>

NOTE: It is extremely helpful for one to recognize these decimal/fraction equivalents. It is very difficult to change repeating decimals to fraction form (and it is beyond the scope of this course).
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimal Fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Numerator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Denominator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lowest Terms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Divisor</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For questions 6 – 17, write the following as mixed numbers or fractions in lowest terms:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>125.25</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>8.08</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.071</td>
<td></td>
</tr>
</tbody>
</table>
For questions 18 – 29, write the following decimals in fraction notation (see if you recognize the common fraction equivalent for the decimal given):

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18)</td>
<td>0.25</td>
<td>19)</td>
</tr>
<tr>
<td>21)</td>
<td>0.875</td>
<td>22)</td>
</tr>
<tr>
<td>24)</td>
<td>0.625</td>
<td>25)</td>
</tr>
<tr>
<td>27)</td>
<td>0.6̅</td>
<td>28)</td>
</tr>
</tbody>
</table>
For questions 30 – 41, write the following fractions in decimal notation (see if you recognize the common decimal equivalent for the fraction given. Hint: 1st write improper fractions as mixed numbers):

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 30 | \[
\frac{11}{3}
\] |   |
| 31 | \[
\frac{5}{2}
\] |   |
| 32 | \[
\frac{11}{4}
\] |   |
| 33 | \[
\frac{10}{3}
\] |   |
| 34 | \[
\frac{9}{4}
\] |   |
| 35 | \[
\frac{1}{5}
\] |   |
| 36 | \[
\frac{1}{8}
\] |   |
| 37 | \[
\frac{1}{9}
\] |   |
| 38 | \[
\frac{3}{8}
\] |   |
| 39 | \[
\frac{7}{8}
\] |   |
| 40 | \[
\frac{11}{10}
\] |   |
| 41 | \[
\frac{5}{8}
\] |   |
Chapter 4.3

ROUNDDING DECIMALS

Rounding Rules for Decimals are the same as the rounding rules for whole numbers with one difference. This happens at the end of the number. Instead of changing all the digits to the right of the line/wall to zeros, you cut off the remaining digits.

To round any Decimal:

1. **LOCATE** the digit in the place value you are rounding to by using the zero trick. The number of zeros in the place value name equals the number of decimal places you are rounding to.

   - 10ths place = 1 decimal place in the rounded number
   - 100ths place = 2 decimal places in the rounded number
   - 1000ths place = 3 decimal places in the rounded number
   - 10,000ths place = 4 decimal places in the rounded number
   
   Etc.

2. **MARK IT** by drawing a vertical line/wall to the right of the place you are rounding.

3. **LOOK** at the digit just to the right of the vertical line/wall. This digit will tell you what to do.

4. **DECIDE:**
   a) If the digit just to the right of the vertical line/wall is: 0, 1, 2, 3, or 4 the entire number on the left of the vertical line/wall stays the same.
   b) If the digit just to the right of the vertical line/wall is: 5, 6, 7, 8, or 9 ADD 1 to the number on the left of the vertical line/wall (increasing the indicated place value by one).

5. **FINISH:** In either case, all the digits to the right of the vertical line/wall are cut off.
Ex.A: Round 1.547 to the nearest hundredth.

<table>
<thead>
<tr>
<th>LOCATE:</th>
<th>MARK IT:</th>
<th>LOOK:</th>
<th>DECIDE:</th>
<th>FINISH:</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are two zeros in the number 100, (for the “100ths” place) so there are two decimal places in the answer.</td>
<td>Draw a vertical line/wall to the right of the second decimal place. (Here, to the right of the digit 4.)</td>
<td>The digit just to the right of the vertical line/wall is a 7.</td>
<td>Since the digit to the right of the wall is a 7, you add 1 to the number on the left of the line/wall: 1.54 becomes 1.55</td>
<td>Cut off all digits to the right of the line/wall. Here it is the digit 7.</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
1.547 & \quad \text{ROUND TO} \quad 1.55
\end{align*}
\]

Answer: 1.547 ≈ 1.55

NOTE: Remember that ≈ stands for “is approximately equal to.”

NOTE: Check to see if the answer has the number of decimal places required by the question.

The same rules apply for rounding repeating decimals. You must first write out the repeating part a few times before rounding.

Ex.B: Round 9.16̅ to the 1000ths place.

<table>
<thead>
<tr>
<th>LOCATE:</th>
<th>MARK IT:</th>
<th>LOOK:</th>
<th>DECIDE:</th>
<th>FINISH:</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are three zeros in the number 1000 (for “1000ths”) so there are three decimal places in the answer.</td>
<td>First be sure to write the repeating part a few times. Then draw a vertical line/wall to the right of the third decimal place.</td>
<td>The digit just to the right of the vertical line/wall is a 6.</td>
<td>Since the digit to the right of the wall is a 6, you add 1 to the number on the left of the wall: 9.166 becomes 9.167</td>
<td>Cut off all digits to the right of the line/wall. Here, cut off the extra sixes.</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
9.166 & \quad \text{ROUND TO} \quad 9.167
\end{align*}
\]

Answer: 9.16̅ ≈ 9.167

Mathematical Foundations by Debra J. Grodchenkik, Ph.D., Jennifer W. Kohut, et al at Nassau Community College is licensed under CC-BY-SA 4.0
Problems:
1) Round 235.09725 to the nearest 10th.

2) Round 235.09725 to the tens place.

3) Change $\frac{5}{32}$ to a decimal and round to the 100ths place.

4) Round 35.97251 to the nearest 1000th.

5) Round 23.593 to the nearest unit or (to the nearest whole number) or (to the 1’s place).

6) Write $\frac{5}{9}$ as a decimal, rounded to 2 decimal places.

7) Round 0. $\overline{28}$ to the 1000ths place.

8) Round 0.2$\overline{8}$ to three decimal places.

9) Round 21.654 to the nearest unit.
ESTIMATING

Remember, for any estimate to **round each number first** and then add, subtract, multiply, or divide. You are not concerned with exact details here, so work with the simple numbers that rounding provides for you!

**To estimate a sum or difference:**

**First round** all numbers to the *same* place, **then add or subtract** to get the estimated sum or difference. If given the choice, round all numbers to the highest *place* that works for most of the numbers. Try to round so that at least ½ of the numbers remain in the problem, that is, they don’t round to zero.

**To estimate a product or quotient:**

First round **each** number to its highest place value (separately) and then multiply or divide.

**Problems:**

1) If you want to buy several grocery items costing $7.29, $2.65, $3.99, $4.18, and $5.50, approximately how much will these items cost?

2) If you want to purchase several items of clothing costing $17.29, $22.95, $37.99, $44, and $65.50, approximately how much will these items cost?
3) If carpet costs $18.95 per sq. yd. and a room measures 8.4 yards by 5.8 yards, what is the approximate cost of this carpeting?

4) If you borrow $8942 and will pay it back in 30 monthly payments, what is the approximate monthly payment?

NOTE: What happens when an exact answer looks funny? For example, if you were solving problem 4 above, looking for an exact answer, what would that answer be? Does it look strange in any way? If so, what should you do about it?
Homework 4.3   Name:

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Place Value Names</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Rounding</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Estimating</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 4 - 8, round the following numbers to the places indicated:

<table>
<thead>
<tr>
<th></th>
<th>Hundredths place</th>
<th>Thousandths place</th>
</tr>
</thead>
<tbody>
<tr>
<td>4) 87.0359</td>
<td>a)</td>
<td>b)</td>
</tr>
<tr>
<td>5) 4.9638</td>
<td>a)</td>
<td>b)</td>
</tr>
<tr>
<td>6) 11.09</td>
<td>a)</td>
<td>b)</td>
</tr>
<tr>
<td>7) 2051.8956</td>
<td>a)</td>
<td>b)</td>
</tr>
<tr>
<td>8) 82.83</td>
<td>a)</td>
<td>b)</td>
</tr>
</tbody>
</table>
For questions 4 – 8 (cont.), round the following numbers to the places indicated:

<table>
<thead>
<tr>
<th></th>
<th>Tenths place</th>
<th>Ones place</th>
<th>Tens place</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>87.0359</td>
<td>c)</td>
<td>d)</td>
</tr>
<tr>
<td>5</td>
<td>4.9638</td>
<td>c)</td>
<td>d)</td>
</tr>
<tr>
<td>6</td>
<td>11.0\bar{9}</td>
<td>c)</td>
<td>d)</td>
</tr>
<tr>
<td>7</td>
<td>2051.8956</td>
<td>c)</td>
<td>d)</td>
</tr>
<tr>
<td>8</td>
<td>82.8\bar{3}</td>
<td>c)</td>
<td>d)</td>
</tr>
</tbody>
</table>

9) If a large popcorn at a movie theater costs $5.89, a large Gulp soda costs $3.78 and a box of Jujubes costs $4.17, about how much will all this cost before tax?

10) An estate worth $748,326 is going to be split evenly among 7 heirs. Approximately how much will each heir inherit?
DECIMAL OPERATIONS

For most decimal work you will be using the calculator, but just in case you don’t have one to use and you must complete problems with decimals in them, here are the pencil and paper rules:

Regardless of how you complete your work, remember to have a good idea if your answers seem reasonable! That is, don’t forget to estimate your answers, especially if you are using the calculator!

Addition and Subtraction of Decimals Without a Calculator:

Line up the decimal points and then add (or subtract, as indicated).

**NOTE:** You can add extra zeros to help line up the place value when adding or subtracting.

Remember, you can only add things that have the same labels, so by lining up the decimal points, you are adding “tens to tens”, “ones to ones”, “tenths to tenths”, “hundredths to hundredths”, etc.

**Ex. A:** \(12.02 + 2.012 + 0.12 + 1.0002 = \) __________  

**Without a calculator:**  
\[
\begin{array}{c}
12.02 \\
2.012 \\
0.12 \\
+ 1.0002 \\
\hline
\end{array} \\
\begin{array}{c}
12.0200 \\
2.0120 \\
0.1200 \\
+ 1.0002 \\
\hline
15.1522
\end{array}
\]

**Estimate:** \(12 + 2 + 0 + 1 = 15\)  
15 is close to the actual answer 15.1522.

**Answer:** 15.1522 is the sum.

**Ex B:** Find the difference between 78.08 and 2.452.  

**Without a calculator:**  
\[
\begin{array}{c}
78.08 \\
- 2.452 \\
\hline
\end{array} \\
\begin{array}{c}
78.080 \\
- 2.452 \\
\hline
75.628
\end{array}
\]

**Estimate:** \(78 – 2 = 76\)  
76 is close to the actual answer 75.628.

**Answer:** 75.628 is the difference.
**Multiplication of Decimals Without a Calculator:**

**Step 1:** Multiply the numbers (factors), ignoring the decimal point.

**Step 2:** Count all of the decimal places in all of the factors combined.

Be sure that the answer (product) has this number of decimal places.

**NOTE:** The estimate can also help you place the decimal point.

**Ex. C:** 17.2 x 0.17 = 

Estimate: 20 x 2 = 40

Without a calculator:

<table>
<thead>
<tr>
<th>Step 1:</th>
<th>17.2</th>
<th>172</th>
<th>17.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 0.17</td>
<td>x 17</td>
<td>1204</td>
<td>1204</td>
</tr>
<tr>
<td></td>
<td>+ 1720</td>
<td>+ 1720</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2924</td>
<td>2.924</td>
<td></td>
</tr>
</tbody>
</table>

Answer: 29.24 is the product.

**Division:**

(Yuck! Long division with decimals is a good reason to have a calculator available!)

**Step 1:** Write your calculator division as long division.

Be sure to place the divisor properly. (Inside ÷ Outside).

**Step 2:** Move the decimal point to the end of the divisor, (outside number) making it a whole number.

**Step 3:** Move the decimal point of the dividend (inside number) the same number of places.

**Step 4:** Immediately, bring the decimal point straight up to the answer (quotient) line.

**Step 5:** Do the long division. Be sure to line up your work and place the decimal point properly.

**Ex. D:** 3.1542 ÷ 15.02

<table>
<thead>
<tr>
<th>Step 1: Inside ÷ Outside</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.02 ) 3.1542</td>
</tr>
<tr>
<td>315.42</td>
</tr>
<tr>
<td>- 3004</td>
</tr>
<tr>
<td>1502</td>
</tr>
<tr>
<td>- 1502</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

Answer: 0.21 is the quotient.
WORD PROBLEMS WITH DECIMAL OPERATIONS

1a) In the figure below, find the length of y and z.

2.75 ft.
1.4 ft.
1.4 ft.
2.75 ft.

9.15 ft.
4.25 ft.

b) Find the area for the figure in question 1a.

2) Find the area of a rectangle, to the nearest tenth of a square yard, if its length is 2.45 yds and its width is 6.49 yds.
3) It costs $10.50 a day plus $0.12 per mile to rent a scooter. How much would it cost to rent the scooter for four days if it is driven a total of 112 miles?

4) Which is the better buy, an 8-pack of gum for $9.52 or a 12 pack for $14.76?

5) A salesperson earns $15.12 per hour for a 40-hour work week and time and a half for each hour of overtime. If Dave worked 48 hours during the week, what was his gross salary for that week?
Homework 4.4  Name:

Before you do your homework please review the terms in this chapter and the method(s) for solving word problems, as well as rounding decimals. If you are not familiar with any or all of these terms or methods, please refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

Use any appropriate method(s) that will help you arrive at reasonable solutions for the following word problems. Remember to show all work & label all answers.

1. Find the values for x and y in the following picture.

   ![Diagram with measurements]

   X = ?
   Y = ?

2. Find the perimeter of the shape shown above in problem number 1.

3. Find the area of the shape in problem 1 above and round your answer to the nearest hundredth of a square inch.
For questions 4 - 7, use the following numbers and for answers with more than 3 decimal places, round to the nearest thousandth: 13.621 and 4.3

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4) Find the sum:</td>
<td>5) Find the difference:</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Find the product:</td>
<td>7) Find the quotient (use 4.3 as the divisor):</td>
</tr>
</tbody>
</table>

8. When you rent a car, the rental company often will base your total cost on two items, the number of days you rent the car and the number of miles you drive the car. Suppose a rental car company charges you $45.98 per day for each day you have the car as well as $0.28 per mile for each mile you drive the car. How much will it cost if you rent a car for 3 days and drive it for a total of 633 miles?
9. Jean, a construction worker, is paid $23.51 for each hour that she works up to and including 40 hours. For any hours she works over the 40 hours in one week she is paid overtime. Overtime is 1.5 times the regular pay rate. If Jean works for 57 hours one week, what will her gross paycheck be?

10. If the Math Lab buys 17 calculators for a total of $239.88 (including tax), what is the unit cost (including tax)?

11. If you can drive 365 miles & use 12.5 gallons of gas, what is your gas mileage (mpg or miles per gallon)?
12. The typical tip you should leave for good service at a restaurant is 1/5 of the subtotal on your bill (you can calculate the tip by multiplying 1/5 times the subtotal on bill). If the subtotal on your dinner bill is $43.78, how much should you leave as a tip if the service was good? NOTE: First convert 1/5 to a decimal and then multiply. Round to the nearest cent (penny) if necessary.

13. A typical purchase order can be found in a restaurant. A party of 8 people goes out for breakfast. Three people order the #1, (which has everything) at $9.75 each. Two people order the pumpkin walnut pancakes at $9.50 each. Two people get the croissant stuffed French toast special at $14.25 each and 8th person gets the veggie, turkey, and egg white omelet for $10.89. All 8 people get coffee at $2.35 each and 6 of the people get juice at $2.99 each.

A) What was the total of the bill before tax and tip (this is the subtotal)?
B) If they split the bill evenly, what is each person’s share?
(Bonus) If the service was great, how much would you tip?

Don’t worry about tax for this problem. Use the table below to organize your data.

*See page 149 for unit price information.

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost per item (Unit Price*)</th>
<th># ordered</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 special</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pumpkin Walnut Pancakes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Croissant Stuffed French Toast</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Veg, Turk, Egg White Omelet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coffee</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Juice</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 4.5

ORDER OF OPERATIONS WITH DECIMALS

Recall the rules for Order of Operations:

\[ P \quad E \quad M/D \quad A/S \]

- **P**: Work the problem inside the Parentheses 1st to get a single value. Note: if there is no operation to complete inside, it means multiply:
  
  Ex. (2)(3) or 2(3) means 2 \times 3.

- **E**: Evaluate Exponents and roots next.

- **M/D**: Means Multiplication and/or Division, as they appear in the problem moving from left to right through the expression.

- **A/S**: Means Addition and/or Subtraction, as they appear in the problem moving from left to right through the expression. Remember: Add and Subtract LAST from left to right.

These rules apply to any kind of numbers. They even apply when you don’t have numbers, for example in algebra or in statistical formulas.

If you are working with decimals, you will probably be using the calculator.

Evaluate the following problems:

1) \((2.1)(3.4) - 1.9 + 2.5\)

2) \(13.33 \div (0.5 + 3.8)\)

3) \((2.3)^2 - 0.5 + 1.1 \div 0.55\)

4) \((3.1)^2 \div 0.8 + 0.06\)
ORDER OF OPERATIONS WITH DECIMALS AND FRACTIONS

If you have problems that contain **decimals as well as fractions**, it can be easier if you change all of your fractions to decimals. Then you can use the calculator, but...

IF you have a fraction that becomes a repeating decimal, it is impossible to put the repeating decimal into the calculator.

So, start working with the fractions and see what happens. Sometimes the repeater “disappears” because of how the problem works.

Problems:

1) \( \left( \frac{2}{3} \right) \left( \frac{3}{4} \right) \div 0.05 \)

2) \( \frac{1}{2} + 12.2 - (3.5)(2.4) \)

3) \( 3.6 \div \frac{1}{4} - 10 \frac{1}{5} + 7.3 \)

4) \( \left( \frac{1}{4} \right)^2 + 2.8 + (2.5)^2 \)

5) \( 14.6 \div \left( \frac{1}{2} \right)^2 + 6.9 \)

6) \( \frac{7}{8} (1.12) + 4.05 \)
**COMPARING DECIMALS**

Suppose you need to choose the larger or smaller of two numbers, or list the numbers in size order. This is very easy IF you have whole numbers to work with.

What if you have decimals?

This can also be easy if those decimals have a **non-zero** whole number part. If so, just compare the whole number side and you are done.

What if the whole numbers are all zero OR some of the whole numbers are the same?

Then you need to compare the decimals to each other. You must be very careful when doing this!

**To compare decimal numbers:**

**Step 1:** Compare whole numbers. If the whole numbers are the same go on to step 2.

**Step 2:** Line up the decimal numbers vertically with the decimal points underneath each other in a column.

**Step 3:** Add enough zeros to the right of a terminating decimal so that all the numbers have the same number of decimal places. If the number is a repeating decimal then add enough of the repeating digit(s) to make the same number of decimal places.

**Step 4:** Look to the digit in the first decimal place and compare 10ths to 10ths. If the digits in the 10ths places are the same, move one more place into the decimal side and compare 100ths to 100ths, etc., until you can decide which number is larger or smaller.

**Ex. A:** List the following decimals from smallest to largest:

0.3 \hspace{1em} 0.33 \hspace{1em} 0.033

<table>
<thead>
<tr>
<th>Align the numbers vertically.</th>
<th>Add extra 0’s or repeating digits.</th>
<th>Compare the 10ths place digit. 0.033 is the smallest.</th>
<th>Compare the 100ths place digit of the remaining numbers they are the same. Compare the 1000ths place digit. 0.330 is the next smallest.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
</tr>
<tr>
<td>0.33</td>
<td>0.330</td>
<td>0.330</td>
<td>0.330</td>
</tr>
<tr>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
</tr>
</tbody>
</table>

**Answer:** 0.033, 0.33, 0.3 are listed from smallest to largest.

NOTE: Always make your final list using the numbers given in the original problem.
List the following from smallest to largest:

1) \(0.8\)  
2) \(0.07\)  
3) \(10.09\)  
4) \(0.82\)  
5) \(4.7\)  
6) \(10.9\)  
7) \(0.802\)  
8) \(4.07\)  
9) \(10.49\)  
10) \(4.6\)  
11) \(10.409\)  
12) \(0.7\)  
13) \(0.49\)

**COMPARING DECIMALS AND FRACTIONS**

Before you compare fractions and decimals, you must first change all fractions to decimals, then follow the steps outlined above.

1) 0.76  
2) 0.53  
3) \(\frac{1}{3}\)  

\[7.6\quad 5\frac{1}{3}\] 

use \(<\) or \(=\) to fill the box

\[
\frac{3}{4}\quad \frac{1}{2}\quad 0.15\quad \frac{3}{5}
\]

4) Based on the information below, which brand of soup is the better buy?

<table>
<thead>
<tr>
<th>Brand</th>
<th>Number of ounces per can</th>
<th>Price per can</th>
<th>Unit price (price per oz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liptan</td>
<td>17</td>
<td>51¢</td>
<td></td>
</tr>
<tr>
<td>Campbell’s</td>
<td>12.4</td>
<td>80¢</td>
<td></td>
</tr>
<tr>
<td>Progressa</td>
<td>13</td>
<td>69¢</td>
<td></td>
</tr>
</tbody>
</table>
Homework 4.5      Name:

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Order of Operations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Changing Fractions to Decimals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Changing Decimals to Fractions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Comparing Decimals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Better Buy (Better Value)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For questions 6 - 13, perform the indicated operations:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6)</td>
<td>2.5 x 2 ÷ 11</td>
<td>7)</td>
<td>(17.36 ÷ 3.1) ÷ 2.8</td>
</tr>
<tr>
<td>8)</td>
<td>(2.2)^2 - 2(0.4) ÷ 1/4</td>
<td>9)</td>
<td>( \frac{1}{3} (3.9) + (5.6 - 4.9)^2 )</td>
</tr>
<tr>
<td>10)</td>
<td>( \left( \frac{1}{2} + 0.75 \times 2.4 \right) ÷ \sqrt{\frac{1}{4} - \frac{3}{5}} )</td>
<td>11)</td>
<td>[ \left( \frac{1}{3} \right) \left( \frac{6}{5} \right) + 2.1 ] ÷ (0.5)^2</td>
</tr>
<tr>
<td>12)</td>
<td>[1.2 \times 5.5 - (0.6)^2] ÷ 6 - 0.2^2</td>
<td>13)</td>
<td>2.4 ÷ \left( \frac{1}{2} \right)^2 - \sqrt{4} + (0.6)^2</td>
</tr>
</tbody>
</table>
For questions 14 - 17, put the following in size order, from smallest to largest:

<table>
<thead>
<tr>
<th>Question</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>14)</td>
<td>0.0612</td>
</tr>
<tr>
<td>15)</td>
<td>0.0612</td>
</tr>
<tr>
<td>16)</td>
<td>1/8</td>
</tr>
<tr>
<td>17)</td>
<td>1/8</td>
</tr>
</tbody>
</table>

For questions 18 - 21, use <, > or = to compare the following numbers:

<table>
<thead>
<tr>
<th>Question</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>18)</td>
<td>Compare 12.0010324 to 12.0100324</td>
</tr>
<tr>
<td>19)</td>
<td>Compare 0.1205 to 0.005006</td>
</tr>
<tr>
<td>20)</td>
<td>Compare 1/9 to 0.1</td>
</tr>
<tr>
<td>21)</td>
<td>Compare 1/27 to 0.37</td>
</tr>
</tbody>
</table>
22. Which size box of Cheerios is a better buy, a 15 oz box for $4.29, a 12 oz box for $3.29, or an 18 oz box for $5.29?

23. Ivory bar soap comes in several different size packages. At a local drug store, the following are available:

- A 4-pack sells for $4.29 and each bar weighs 4 ounces.
- Single bars, each weighing 4 ounces, cost $1.
- A 3-pack sells for $2.19 and each bar weighs 3.1 ounces.

What is the unit price for each available package? Which package should you buy to get the best value for your money? NOTE: Is the price per bar helpful here?
**Area**

*Definition:* Area measures the size of a flat surface using square units. The formula for the area of a rectangle:

\[
\text{Total # of squares} = \text{# of squares per row} \times \text{# of rows}
\]

*Example:* 4 squares/row x 3 rows = 12 squares

This rectangle is 12 *square* units

**Associative Law of Multiplication**

*Definition:* Regrouping more than 2 numbers results in identical products. (Number order remains the same.)

*Example:* 
\[(1 \times 2) \times 3 = 1 \times (2 \times 3),\] regrouping numbers will not change the final product.

**Base**

*Definition:* The number being repeatedly multiplied in an exponential expression.

*Example:* In the exponential expression, \(5^4\), the base is 5, the repeated factor.

**Commutative Law of Multiplication**

*Definition:* The product of 2 numbers is the same regardless of number order.

*Example:* \(3 \times 5 = 5 \times 3\), switching (commuting) the order of the numbers doesn't change the final product.

**Cross Products**

*Definition:* They are the products created when cross multiplying two fractions.

*Example:* 
\[
\frac{4}{5} = \frac{8}{10}
\]

4 x 10 = 5 x 8

40 and 40 are the cross products.

**Decimal-Fraction**

*Definition:* A decimal-fraction is a fraction with the denominator equal to a power of ten.

*Example:* 
\[
\frac{0}{10}, \frac{0}{100}, \frac{0}{1000}, \frac{0}{10000}, \text{ Etc.}
\]

**Denominator**

*Definition:* The bottom number in a fraction, which represents the total number of equal size portions you break one whole thing into.

*Example:* If the denominator is 4, break the whole thing into 4 equal size portions.

\[
\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4}
\]

**Divisor**

*Definition:* A divisor is the number that is being repeatedly subtracted from an original number to determine how many complete groups are contained in the original number.

*Example:* In the division sentence: 
\(12 \div 3 = 4\), 3 is the divisor and there are 4 full groups of 3 contained in the number 12.

**Fraction**

*Definition:* A fraction is a portion of a whole thing.

*Example:* 
\[
\frac{N}{D} = \frac{\text{Any Whole Number}}{\text{Any Whole Number, except zero}}
\]

**Numerator**

*Definition:* The top number in a fraction, which represents the number of equal size portions you want.

*Example:* If the numerator is 3 then you want three portions. (In other words, shade three portions.)

\[
\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4}
\]
**Proportion**

**Definition:** Two ratios form a proportion if their cross products are equal.

**Example:**
\[
\frac{4}{5} = \frac{8}{10}
\]
\[
4 \times 10 = 5 \times 8
\]
\[
40 = 40
\]

Since \(40 = 40\), the ratios \(\frac{4}{5}\) and \(\frac{8}{10}\) are proportional.

---

**Standard Notation**

**Definition:** A number written using digits.

**Example:** 20.76 is a number written in standard notation.

---

**Word Names**

**Definition:** A number written using words.

**Example:** A word name for 2,000,305,076 is two billion, three hundred five thousand, seventy-six.

---

**Chapter 4 Important Ideas and Concepts**

**Better Buy:** Any item identical in quantity and quality to another, which costs less, is a Better Buy.  
*See pages 149, 184, and 356.*

**Comparing Decimals:** Line up the decimal points in the numbers and going from left to right find the first column where the digits are different and start your comparison there.  
*See pages 355 - 356.*

**Cross Multiplying:** To cross multiply we multiply the numerator of one fraction by the denominator of the other fraction only across an equal sign.  
*See pages 216, 217, 253, and 297.*

**Estimate:** To find an approximate answer to any mathematical problem by first rounding all numbers and then performing the indicated operation with the rounded numbers.  
*See pages 39, 42, 80, 104, 185, and 340.*

**Fraction/Decimal Conversion**

All fractions can be written in decimal form by dividing the numerator by the denominator. All decimals can be changed to fractions by naming the decimal and writing it in fraction (mixed number) form. REMEMBER TO REDUCE THE FRACTION TO LOWEST TERMS!  
**Example:**

\[
\frac{3}{4} = 3 \div 4 = 0.75, \text{ so, } \frac{3}{4} = 0.75
\]

0.75 = “seventy-five hundredths” = \(\frac{75}{100} = \frac{3}{4}\)

*See pages 325, 327, 330, 331.*
Order of Operations aka P E (M/D) (A/S): Multi-step problems with many different operations are to be done in a specific order.

Step 1: P stands for (Parentheses), [brackets], or {braces} - (grouping symbols) - Any mathematical operations inside grouping symbols are evaluated first.

Example: \(8 \times (3 + 0.1) = 8 \times 3.1 = 24.8\)

Step 2: E stands for Exponents - Any numbers with exponents (including square root symbols) are done second.

Example: \(4 + 0.2^3 = 4 + 0.008 = 4.008\) \(5 + \sqrt{0.36} = 5 + 0.6 = 5.6\)

Step 3: M/D stands for Multiplication and Division - Any multiplication and division problems are done third, working in order from left to right. If you see multiplication first (on the left) do that, but if division is first (on the left) do that.

Example: \(42 \div 7 \times 6 = 6 \times 6 = 36\) \(3 \times 4 \div 2 = 12 \div 2 = 6\)

Step 4: A/S stands for Addition and Subtraction – Adding and Subtracting are done last, working in order from left to right. If you see addition first (on the left) do that, but if subtraction is first (on the left) do that.

Example: \(11 - 3 + 8 = 8 + 8 = 16\) \(5 + 6 - 11 = 11 - 11 = 0\)

See pages 175, 186, and 353.

PEMDAS: See Order of Operations above.

Reducing Fractions to Lowest Terms: A fraction is in lowest terms when the numerator and the denominator have no prime factors in common. To reduce a fraction algebraically, remember to Factor, Cancel, Multiply.

See pages 219 – 220 and 297.

Rounding: Determining an approximate value of a number based on its proximity to numbers above and below it in a specific place value.

See pages 39 – 40 and 337.

Unit Cost: Unit cost is found by the formula:

\[
\text{Total cost} \div \text{Total Number of Units} = \text{Cost for One Unit}
\]

See page 149 and 186.
For questions 1 – 3, give the name of the place value for the digit 7 in the following numbers:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>0.0738</td>
<td>2)</td>
</tr>
</tbody>
</table>

For questions 4 – 9, write the word names for the following numbers:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4)</td>
<td>28.2</td>
<td>5)</td>
</tr>
<tr>
<td>6)</td>
<td>0.0003</td>
<td>7)</td>
</tr>
<tr>
<td>8)</td>
<td>375.38</td>
<td>9)</td>
</tr>
</tbody>
</table>
For questions 10 – 12, in 4327.5619 give the digit that represents the following place values:

<table>
<thead>
<tr>
<th>10) tenths place</th>
<th>11) ten thousandths place</th>
<th>12) hundreds place</th>
</tr>
</thead>
</table>

For questions 13 – 18, write the following word names in standard (decimal) notation:

<table>
<thead>
<tr>
<th>13) Four and five tenths</th>
<th>14) One thousand ninety-seven ten thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>15) Seventy and one hundredth</td>
<td>16) Fifty-one ten thousandths</td>
</tr>
<tr>
<td>17) One and fourteen hundred thousandths</td>
<td>18) Three hundredths</td>
</tr>
</tbody>
</table>
For questions 1 – 12, write the following as mixed numbers or fractions in lowest terms:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>0.55</td>
<td>2)</td>
</tr>
<tr>
<td>4)</td>
<td>11.37</td>
<td>5)</td>
</tr>
<tr>
<td>7)</td>
<td>5.04</td>
<td>8)</td>
</tr>
<tr>
<td>10)</td>
<td>0.132</td>
<td>11)</td>
</tr>
</tbody>
</table>
For questions 13 – 24, write the following numbers in standard (decimal) notation:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13)</td>
<td>$\frac{5}{9}$</td>
<td></td>
</tr>
<tr>
<td>14)</td>
<td>$\frac{3}{20}$</td>
<td>$\frac{14}{15}$</td>
</tr>
<tr>
<td>15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16)</td>
<td>$\frac{13}{80}$</td>
<td></td>
</tr>
<tr>
<td>17)</td>
<td>$\frac{11}{40}$</td>
<td>$\frac{9}{100}$</td>
</tr>
<tr>
<td>18)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19)</td>
<td>$\frac{1}{6}$</td>
<td></td>
</tr>
<tr>
<td>20)</td>
<td>$\frac{6}{11}$</td>
<td></td>
</tr>
<tr>
<td>21)</td>
<td></td>
<td>$\frac{9}{8}$</td>
</tr>
<tr>
<td>22)</td>
<td>$\frac{28}{111}$</td>
<td></td>
</tr>
<tr>
<td>23)</td>
<td>$\frac{19}{60}$</td>
<td></td>
</tr>
<tr>
<td>24)</td>
<td>$\frac{9}{50}$</td>
<td></td>
</tr>
</tbody>
</table>
For questions 1 - 5, round the following numbers to the places indicated:

<table>
<thead>
<tr>
<th></th>
<th>100ths place</th>
<th>1000ths place</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 24.9567</td>
<td>a)</td>
<td>b)</td>
</tr>
<tr>
<td>2) 4298.0123</td>
<td>a)</td>
<td>b)</td>
</tr>
<tr>
<td>3) 17.1982</td>
<td>a)</td>
<td>b)</td>
</tr>
<tr>
<td>4) 87.2134</td>
<td>a)</td>
<td>b)</td>
</tr>
<tr>
<td>5) 198.345</td>
<td>a)</td>
<td>b)</td>
</tr>
</tbody>
</table>

For questions 1 – 5 (cont.), round the following numbers to the places indicated:

<table>
<thead>
<tr>
<th></th>
<th>10ths place</th>
<th>1s place</th>
<th>10s place</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 24.9567</td>
<td>c)</td>
<td>d)</td>
<td>e)</td>
</tr>
<tr>
<td>2) 4298.0123</td>
<td>c)</td>
<td>d)</td>
<td>e)</td>
</tr>
<tr>
<td>3) 17.1982</td>
<td>c)</td>
<td>d)</td>
<td>e)</td>
</tr>
<tr>
<td>4) 87.2134</td>
<td>c)</td>
<td>d)</td>
<td>e)</td>
</tr>
<tr>
<td>5) 198.345</td>
<td>c)</td>
<td>d)</td>
<td>e)</td>
</tr>
</tbody>
</table>

6) The check at a restaurant was $188.48 including tax and tip. If the bill is to be split evenly amongst 10 people, about how much does each person owe?

7) If you buy one pizza for $14.99, a 2 liter soda is $1.79, garlic knots which are 6 for $1.75 and 6 zeppoles for $3.75, estimate the total cost by rounding to the nearest whole dollar.
1. Find the value for $x$ in the following picture:

2. Find the perimeter of the shape shown above in problem number 1.

3. Find the area of the shape shown above in problem number 1.
For questions 4-7, use the following numbers and for any answer with more than 3 decimal places, round to the nearest thousandth: 483.03 and 24.65:

4) Find the sum: 5) Find the difference:

6) Find the product: 7) Find the quotient (use 24.65 as the divisor):

8) When you go to a lawyer the first visit costs $250. After that, for every hour your lawyer spends working on your case, he/she charges $325 per hour. What will it cost you for your initial visit and 7.5 additional hours of their time?

9) Suppose one is paid time-and-a-half for overtime (that is, 1.5 times the regular hourly pay rate for every hour someone works over 40 hours in one week). If the regular hourly rate of pay for a particular job is $24.46 per hour, A) what would the hourly overtime pay rate be for that job? B) If someone worked 48 hours in one week, what would his/her gross paycheck be? (Gross pay = total pay before any deductions are taken out.)
10) In electronics, measurements need to be accurate to within a hundred thousandth of an inch. If the measurement for a space between two electronic components is supposed to be 0.00081 inches and the actual measurement is 0.00057 inches, how far off are the measurements?

11) My odometer read 52,312.8 at the start of a trip and 52,763.6 at the end of the trip. If I used 18.4 gallons of gas, what was my gas mileage (how many miles per gallon did I get) for the trip?

12) 15% is considered an okay tip while 20% is considered a generous tip for very good service. What would be the average of the two tip percentages?
For questions 1 – 8, perform the indicated operations:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>$\sqrt{\left(\frac{1}{3}\right)^2 (9)^2}$</td>
</tr>
<tr>
<td>2)</td>
<td>$3.5 \times 3 \times \frac{1}{7}$</td>
</tr>
<tr>
<td>3)</td>
<td>$3.5 + (3)\left(\frac{1}{2}\right)^2$</td>
</tr>
<tr>
<td>4)</td>
<td>$(4.2)^2 \div (2 \times 0.25) + 0.895$</td>
</tr>
<tr>
<td>5)</td>
<td>$\left(\frac{1}{5} + 0.75 \times 3.6\right) \div \sqrt{\frac{1}{16} + \frac{2}{5}}$</td>
</tr>
<tr>
<td>6)</td>
<td>$\left[\left(\frac{1}{8}\right)\left(\frac{24}{5}\right) - 0.1\right] \div \left(\frac{1}{4}\right)^2$</td>
</tr>
</tbody>
</table>
For questions 9 - 12, put the following numbers in size order, from smallest to largest:

9)  
- 6.012  
- 0.612  
- 2.162  
- 6.12  
- 0.0612

10)  
- 0.246  
- 0.26  
- 0.126  
- 0.42  
- 0.00621

11)  
- 1/4  
- 0.4  
- 1/2  
- 0.65  
- 3/5

12)  
- 7.5  
- 3/4  
- 7.05  
- 4/5  
- 0.85
For questions 13 - 16, use <, > or = to compare the following numbers:

<table>
<thead>
<tr>
<th>13) Compare 2.000103 to 2.0103</th>
<th>14) Compare 0.205 to 0.05006</th>
</tr>
</thead>
<tbody>
<tr>
<td>15) Compare 1/6 to 0.16</td>
<td>16) Compare 1/37 to 0.27</td>
</tr>
</tbody>
</table>

17) A popular discount store has Cheerios on sale for $3.68 for the 17 oz. box, $3.98 for the 21.6 oz. box and $4.58 for the 26.6 oz. box. Which size box of Cheerios is the better value?

18) The single-serve cups of Cheerios (1.83 oz. each) are available for $9.99 for a 6 pack.
   A) How much do these cost per ounce?
   B) If this were the price per ounce in the large box of Cheerios, what would the 26.6 oz box cost?
   C) Is the price of the single serving cups worth the convenience?
4.1 extra practice answers:

1) hundredths  
2) hundreds  
3) ten thousandths  
4) twenty-eight and two tenths  
5) sixteen and seventy-five thousandths  
6) three ten thousandths  
7) four and five hundred sixty-one thousandths  
8) three hundred seventy-five and thirty-eight hundredths  
9) three and seventy thousand five hundred thirty-eight millionths  

10) 5  
11) 9  
12) 3  
13) 4.5  
14) 0.1097  
15) 70.01  
16) 0.0051  
17) 1.00014  
18) 0.03

4.2 extra practice answers:

1) \(\frac{11}{20}\)  
2) \(\frac{2}{125}\)  
3) \(\frac{523}{1000}\)  
4) \(\frac{1137}{100}\)  
5) \(\frac{1}{5}\)  
6) \(\frac{21}{50}\)  

7) \(\frac{5}{25}\)  
8) \(\frac{23}{500}\)  
9) \(\frac{1121}{25}\)  
10) \(\frac{33}{250}\)  
11) \(\frac{27}{5000}\)  
12) \(\frac{75127}{200}\)  

13) \(0.5\)  
14) 2.15  
15) 0.93  
16) 21.1625  
17) 1.275  
18) 0.09  

19) 16.16  
20) \(0.\overline{54}\)  
21) 1.125  
22) \(14.\overline{252}\)  
23) 0.316  
24) 0.18

4.3 extra practice answers:

1a) 24.96  
1b) 24.957  
1c) 25.0  
1d) 25  
1e) 20  
2a) 4298.01  
2b) 4298.012  
2c) 4298.0  
2d) 4298  
2e) 4300  
3a) 17.20  
3b) 17.198  
3c) 17.2  
3d) 17  
3e) 20  
4a) 87.21  
4b) 87.213  
4c) 87.2  
4d) 87  
4e) 90  
5a) 198.35  
5b) 198.345  
5c) 198.3  
5d) 198  
5e) 200

6) $20  
7) $23
4.4 extra practice answers:

1) 3.2 cm.  
2) 56.6 cm.  
3) 147.84 sq. cm.  
4) 507.69

5) 458.38  
6) 11,906.690  
7) 19.596  
8) $2687.50

9A) $36.69/hr  
9B) $1271.92  
10) 0.00024 inches  
11) 24.5 mpg  
12) 17.5%

4.5 extra practice answers:

1) 3  
2) 1.5 or $1\frac{1}{2}$ or 3/2  
3) 4.25 or $4\frac{1}{4}$ or 17/4

4) 36.175  
5) 12  
6) 8  
7) 2

8) 9.15 or $9\frac{3}{20}$ or 183/20  
9) 0.0612; 0.612; 2.162; 6.012; 6.12

10) 0.00621; 0.126; 0.246; 0.26; 0.42  
11) 1/4; 0.4; 1/2; 3/5; 0.65

12) 3/4; 4/5; 0.85; 7.05; 7.5  
13) 2.0001003 < 2.0103

14) 0.205 > 0.05006  
15) 1/6 > 0.16

16) $1/37 < 0.27$  
17) The 26.6 oz. box

18A) $0.91/oz.  
18B) $24.21  
18C) Definitely NOT
CHAPTER 5

PROBLEM SOLVING

WITH

PERCENTS
Solving one step multiplication and division word problems can be challenging. There are three possible ways to work the problem when trying to get the correct answer. (See below.) This gives you a 1 in 3 chance of selecting the correct procedure to get the correct answer.

**RATIOS AND PROPORTIONS**

One Step Multiplication or Division Word Problems

- Multiply 1st number x 2nd number
- Divide 1st number by 2nd number
- Divide 2nd number by 1st number

**Ratios and Proportions** are very useful for solving ONE STEP multiplication and division word problems. If you can find one ratio in the word problem and set it up, you can use that first ratio to set up the second ratio. You have now created a proportion: two equal ratios.

Write the two ratios and set them equal to each other.

Then use the rule you learned in Chapter 3 that says two fractions (ratios) are equal if their cross products are equal. Cross multiply the ratios (set the two cross products equal) and you can solve for the unknown quantity in the proportion.

The beauty of ratios and proportions is you can put any type of number (whole numbers, fractions, or decimals) in the numerator or denominator. It’s the labels associated with those numbers that help one set up the proportion and ultimately solve the problem correctly.

A ratio compares two numbers (two numbers that go together). You may know both numbers (with their corresponding labels) or one of the two numbers may be the unknown quantity.

**Ex. A:** If there are 7 male students and 4 female students in the same class, write the ratio of male to female students in three different ways.

- 7 males to 4 females (Sentence Style)
- 7 males : 4 females (Colon Form)
- \(\frac{7}{4}\) males females (Fraction Form)

**NOTE:** In ratios, numbers are always followed by their labels.

**NOTE:** The fraction form is used most often for proportions.
**Ex. B:** Write the expression “my car gets 15 miles per gallon” as a ratio.

**Numerical Information:**
15 miles
The singular form of the word gallon implies 1 gallon; the word “per” means for each **one**.

\[
\frac{15 \text{ miles}}{1 \text{ gallon}}
\]

**Answer:** The expression “my car gets 15 miles per gallon” written as a ratio is \( \frac{15 \text{ miles}}{1 \text{ gallon}} \)

**Ex. C:** Write the following as a ratio: “How many pieces can be cut from a wire that is 80 meters long?”

**Numerical Information:**
\( N \) = the number of pieces and is also the unknown
80 meters is the length of the wire that is being cut

\[
\frac{N \text{ pieces}}{80 \text{ meters}}
\]

**Answer:** The expression, “How many pieces can be cut from a wire that is 80 meters long,” written as a ratio is \( \frac{N \text{ pieces}}{80 \text{ meters}} \)

Write the following expressions as ratios.

1) The New York State speed limit is 55 miles per hour.

2) How many hours will it take to drive 137.5 miles?

3) Each piece of yarn is \( \frac{2}{3} \) yd. long.

4) How many pieces of yarn can be cut from 30 yards?

5) 3 out of 5 Nassau Community College students use public transportation.

6) If 12,000 students use public transportation how many students are there at NCC?
PROPORTIONS

A proportion is two equal ratios.

Ex. A: Suppose you were considering transferring to a school that had a male to female student ratio of 7 to 4 (7 males to every 4 females). If you had a friend at that school who told you that he has 28 women in his chemistry class, how many men would you expect there to be in that class?

NOTE: The fraction form is used most often for proportions.

Numerical Information:

7 males to every 4 females is the first ratio, use the fraction form and write the word labels.
28 women in the chemistry class
N = the number of men in the chemistry class

\[
\frac{7 \text{ males}}{4 \text{ females}} = \frac{N \text{ men (males)}}{28 \text{ women (females)}}
\]

To Cross Multiply you must find two cross products:
Multiply the numerator of one fraction by the denominator of the other fraction. Do this diagonally in both directions.
Set the two cross products equal to each other and solve for N.

\[
\frac{7}{4} = \frac{N}{28}
\]

\[
4N = 7 \times 28
\]

\[
\frac{4N}{4} = \frac{196}{4}
\]

\[
N = 49 \text{ males}
\]

Answer: There are 49 men in the classroom.

BONUS: How many students are there in his class?

Numerical Information:

28 women in the chemistry class
49 men in the chemistry class

\[
28 \text{ women} + 49 \text{ men} = 77 \text{ students in the class.}
\]

Answer: There are 77 students in the chemistry class.

Solve the following word problems using ratios and proportions.
1) The New York State speed limit is 55 miles per hour. How many hours will it take to drive 137.5 miles?

2) Each piece of yarn is \(\frac{2}{3}\) yd. long. How many pieces of yarn can be cut from 30 yards of yarn?

3) 3 out of 5 Nassau Community College students use public transportation. If 12,000 students use public transportation how many students are there at NCC?
Homework 5.1  Name:

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Proportion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Cross Multiply</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Cross Product</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 5 - 6, consider the following information:

There are 17 students in the class. Nine students are female and 8 are male.

Write the following in fraction form:

5) What fraction of the class is female students?  6) What fraction of the class is male students?
For questions 7 - 8, consider the following information:

There are 17 students in the class. Nine students are female and 8 are male.

Write the following ratios in a) word form, b) colon form and c) fraction form:

<table>
<thead>
<tr>
<th>7) What is the ratio of female students to male students?</th>
<th>8) What is the ratio of male students to female students?</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>9) It takes 8 cups of flour to make a 2 loaves of bread.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) What is the ratio of flour to loaves of bread?</td>
</tr>
<tr>
<td>b) Reduce the above ratio to a unit ratio. In other words, what is the ratio of flour to 1 loaf of bread?</td>
</tr>
</tbody>
</table>

For questions 11-13, are the following ratios proportional? Please explain why or why not:

| 10) \( \frac{4}{7} = \frac{16}{49} \) | 11) \( \frac{3\frac{1}{2}}{4} = \frac{10\frac{1}{2}}{12} \) | 12) \( \frac{4.2}{3.5} = \frac{29.4}{24.5} \) |

---

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Chapter 5.2

PERCENT

The word percent means “per 100” or “out of 100” or “÷ 100”

If you received a grade of 82% on a test, it means you earned 82 points out of 100 points or \( \frac{82}{100} \)

\[ \frac{82}{100} \text{ can be reduced to } \frac{41}{50} \text{ or changed to a decimal by dividing } 82 \div 100 = 0.82 \]

FRACTION \( \leftrightarrow \) DECIMAL \( \leftrightarrow \) PERCENT CONVERSIONS

NOTE: Remembering how to convert between fraction and decimal notation will be very helpful when working with percent conversions. (See Chapter 4.)

WRITING DECIMALS IN FRACTION NOTATION

1) Name the decimal (use the zero trick).
2) Write it as a fraction (or mixed number)
3) REDUCE

Recall: \( 0.084 = \frac{84}{1000} \div 2 = \frac{42}{500} \div 2 = \frac{21}{250} \)

WRITING FRACTIONS IN DECIMAL NOTATION

Remember that a fraction is also notation for division.
Use the calculator (top divided by bottom) and you will get your decimal notation!

Recall: A fraction is just another notation for division \( \frac{21}{50} = 21 \div 50 = 0.42 \)

NOTE: Without a calculator, just do the long division until the division ends or you get a pattern (or use the decimal fraction trick from Chapter 4).
CONVERTING PERCENTS TO NON PERCENT NUMBERS

The one thing to remember any time you see a percent sign is that:

\[ P \text{ “per cent” means “P over 100” so, } P \% = \frac{P}{100} \]

In other words, a % number can be written over 100:

\[
8\% = \frac{8}{100} \\
452\% = \frac{452}{100} \\
17.3\% = \frac{17.3}{100} \\
3\frac{1}{2}\% = \frac{3\frac{1}{2}}{100}
\]

This is only the first step when converting to non-percent notation. Always check to see if anything further must be done to make the answer sound reasonable. Also check to see which notation (form) is required for the answer (decimal, fraction, etc.).

Every number written with a percent sign can be written as an equivalent non-percent

<table>
<thead>
<tr>
<th>Percent Numbers</th>
<th>Non Percent Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P% = \frac{P}{100} )</td>
<td>Decimals</td>
</tr>
<tr>
<td></td>
<td>Whole numbers</td>
</tr>
<tr>
<td></td>
<td>Mixed Numbers</td>
</tr>
<tr>
<td></td>
<td>Fractions</td>
</tr>
</tbody>
</table>

Convert the following numbers to non-percent numbers:

**Ex. A:** Write 8% in fraction and decimal notation:

\[
8\% = \frac{8}{100} = \frac{2 \times 2 \times 2}{2 \times 2 \times 5 \times 5} = \frac{2}{25}
\]

This fraction MUST be reduced to lowest terms.

Answer: 8% = \(\frac{2}{25}\) as a fraction in lowest terms.

To write 8% as a decimal 8% = \(\frac{8}{100}\) = 8 ÷ 100 = 0.08

Answer: 8% = 0.08 in decimal notation.
**Ex. B**: Write 452% as a decimal and as a fraction (or mixed number):

To write 452% as a decimal:

\[
452\% = \frac{452}{100} = 4.52
\]

**Answer**: 452% = 4.52 in decimal notation.

Now to write 452% as a fraction or mixed number:

\[
4.52 = \frac{452}{100} = \frac{26}{50} = \frac{13}{25}
\]

**Use the decimal equivalent of 452%**:
Write it as a mixed number. The proper fraction part is much easier to simplify than the improper fraction you get from the original 452% written as a fraction.

**Answer**: 452% = \(4 \frac{13}{25}\) as a mixed number and

\[
452\% = \frac{113}{25}
\]

as a fraction (an improper fraction) in lowest terms.

**Ex. C**: Write 17.3% as a decimal and as a fraction or mixed number = \(\frac{17.3}{100}\)

To write 17.3% as a decimal:

\[
17.3\% = \frac{17.3}{100} = 17.3 \div 100 = 0.173
\]

**Answer**: 17.3% = 0.173 in decimal notation and \(17.3\% = \frac{173}{1000}\) in * fraction notation.

*Remember that 0.173 has 3 decimal places which represents 1000ths for the fraction form.*
Ex D: Write $3\frac{1}{2}$% as a decimal and as a fraction.

To write $3\frac{1}{2}$% as a decimal:

$$3\frac{1}{2}\% = 3.5\% = \frac{3.5}{100} = 3.5 \div 100 = 0.035$$

Answer: $3\frac{1}{2}$% = 0.035 in decimal notation.

Now to write $3\frac{1}{2}$% as a fraction or mixed number:

$$0.035 = \frac{35}{1000} = \frac{35 \div 5}{1000 \div 5} = \frac{7}{200}$$

Use the decimal equivalent of $3\frac{1}{2}$%:
Write it as a fraction. It will be easier to simplify than starting with the $3\frac{1}{2}$%.

When converting a decimal to fraction form, reduce the decimal-fraction to lowest terms by dividing the numerator and the denominator by 2 or 5.

Answer: $3\frac{1}{2}$% = $\frac{7}{200}$ as a fraction in lowest terms.
Write the following as decimals, fractions and/or mixed numbers (non-percent notation).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1) (\frac{3}{8})</td>
<td>2) (\frac{1}{2})</td>
</tr>
<tr>
<td>3) 69.7%</td>
<td>4) (12\frac{3}{4})</td>
</tr>
<tr>
<td>5) 300%</td>
<td>6) (66\frac{2}{3})</td>
</tr>
</tbody>
</table>
Write the following as decimals, fractions and/or mixed numbers (non-percent notation).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7) ( \frac{2}{5} % ) &amp; 8) 26.4%</td>
<td></td>
</tr>
<tr>
<td>9) 250% &amp; 10) ( \frac{1}{20} % )</td>
<td></td>
</tr>
<tr>
<td>11) ( \frac{4}{5} % ) &amp; 12) ( 3\frac{1}{3} % )</td>
<td></td>
</tr>
</tbody>
</table>
CONVERTING NON PERCENT NUMBERS TO PERCENTS

Every type of number can be written as a percent.

<table>
<thead>
<tr>
<th>Non Percent Numbers</th>
<th>Percent Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimals</td>
<td>P% = ( \frac{P}{100} )</td>
</tr>
<tr>
<td>Whole numbers</td>
<td></td>
</tr>
<tr>
<td>Mixed Numbers</td>
<td></td>
</tr>
<tr>
<td>Fractions</td>
<td></td>
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</tbody>
</table>

If any number can represent a percentage (\( N = P\% \)), then that number can be equated with its percent equivalent to get the following proportion:

\[ N = P\% \quad \text{so,} \quad N = \frac{P}{100} \]

a) If you start with a whole number or decimal, put this number over 1 to set up the proportion, then cross multiply.

b) When starting with a fraction, the set up for the proportion is already done, so cross multiply.

c) When starting with a mixed number, first change to an improper fraction. Then the proportion will be set up (as above), so cross multiply and solve.

**Convert the following numbers to percent numbers:**

**Ex. A:** Convert 0.23 to percent notation.

\[ 0.23 = P\% \quad \rightarrow \quad \frac{0.23}{1} = \frac{P}{100} \quad \rightarrow \quad P = 0.23(100) \quad \rightarrow \quad P = 23 \]

Answer: 0.23 = 23% in percent notation.

NOTE: Because \( P \) is over 100, this means \( P \) percent, so \( P = 23 \) here means \( P\% = 23\% \).
**Ex. B:** Convert 4 to percent notation.

\[ 4 = \frac{4}{1} = \frac{P}{100} \rightarrow P = 4(100) \rightarrow P = 400 \]

**Answer:** 4 = 400% in percent notation.

**NOTE:** Because P is over 100, this means P percent, so P = 400 here means P% = 400%

**Ex. C:** Convert \(2\frac{1}{4}\) to percent notation.

\[
2\frac{1}{4} = \frac{9}{4} = \frac{P}{100} \rightarrow 4P = 9(100) \rightarrow \frac{4P}{4} = \frac{900}{4} \rightarrow P = 225
\]

**Answer:** \(2\frac{1}{4} = 225\%\) in percent notation.

**NOTE:** Because P is over 100, this means P percent, so P = 225 here means P% = 225%

**Ex. D:** Convert \(\frac{4}{5}\) to percent notation.

\[
\frac{4}{5} = \frac{P}{100} \rightarrow 5P = 4(100) \rightarrow \frac{5P}{5} = \frac{400}{5} \rightarrow P = 80
\]

**Answer:** \(\frac{4}{5} = 80\%\) in percent notation.

**NOTE:** Because P is over 100, this means P percent, so P = 80 here means P% = 80%
Write the following in percent notation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 0.61</td>
<td>2) 4.25</td>
</tr>
<tr>
<td>3) $\frac{3}{8}$</td>
<td>4) 3</td>
</tr>
<tr>
<td>5) $1\frac{3}{5}$</td>
<td>6) $\frac{2}{3}$</td>
</tr>
</tbody>
</table>
Write the following in percent notation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7)</td>
<td>0.804</td>
</tr>
<tr>
<td>8)</td>
<td>0.65</td>
</tr>
<tr>
<td>9)</td>
<td>$\frac{7}{10}$</td>
</tr>
<tr>
<td>10)</td>
<td>4.8</td>
</tr>
<tr>
<td>11)</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>12)</td>
<td>$1\frac{2}{5}$</td>
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<tr>
<td>1) Fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Decimal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Percent</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 4 – 15, complete the chart below. Convert from the given form of the number to both of the other two forms:

<table>
<thead>
<tr>
<th>Fraction or mixed number</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>4) 5/8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) 42.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) 42.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7) 40%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8)</td>
<td>9)</td>
<td></td>
</tr>
</tbody>
</table>
16) In a class 1/4 of my students dropped out for personal reasons and 1/8 of the students were dropped because of their attendance records. Of the remaining students, 0.125 of them were not ready to move on so they need to repeat the course. The remaining 1/2 of the class were not only ready to move on, but they went on to MAT 002.

Give the appropriate percentages for the following categories:

<table>
<thead>
<tr>
<th>Category:</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Dropped for personal reasons:</td>
<td>a)</td>
</tr>
<tr>
<td>b) Dropped by me for attendance:</td>
<td>b)</td>
</tr>
<tr>
<td>c) Completed, but need to repeat the course:</td>
<td>c)</td>
</tr>
<tr>
<td>d) Passed and skipped MAT 001:</td>
<td>d)</td>
</tr>
</tbody>
</table>
PERCENT WORD PROBLEMS

Translation or proportions can be used to solve percent word problems. Proportions are the preferred method to solving word problems because the setup never changes, just the given and missing information. This is very similar to the “fraction of a whole” problems that we saw in chapter 3, which we solved using the picture or proportion methods.

**NOTE:** When using the translation method, proceed very carefully!

**SOLVING BASIC PERCENT WORD PROBLEMS**

**Method 1: Translation**

**Ex. A:** What is 5% of 64.82?

1) Circle the numbers: 5% and 64.82
2) Circle the words: “is” and “of.”
3) Circle whatever is left.
   You should have 5 circles.

Translate the numbers and words in the order they are written:
1) “what” means an unknown quantity or “variable ”
2) “is” means equals
3) 5% remains (you can write 5% as \( \frac{5}{100} \) now or later)
4) “of” means multiply
5) 64.82 remains

Now the problem to solve is:

\[
N = 5\% \times 64.82 \rightarrow N = \frac{5}{100} \times 64.82 \rightarrow N = 0.05 \times 64.82 \rightarrow N = 3.241
\]

**Answer:** 3.241 is 5% of 64.82
Translate then solve the following problems.

1) What is 10% of 579.5?

2) 220% of 30 is what number?

3) 70 is 80% of what number?

4) 40.5% of what number is 48.6?

5) 195 is what percent of 78?

6) 372 is what percent of 310?
In chapter 3 we saw basic word problems worded as “fraction of a whole is part” with the ideas of **part** and **whole** in these problems. These can be solved with proportions. The same thing is true with basic percent word problems.

Just like **fraction of a whole** is part of something, **% of a whole** is part of something.

**Method 2: Proportion**

**Ex. A:** What is 5% of 64.82?

Identify the “part” and the “whole” and set up a **proportion** using the same information when changing between fraction and percent notations: 

\[ \frac{P}{100} = \frac{\text{part}}{\text{whole}} \]

Cross multiply to solve for \( N \): 

\[ 100N = 5 \times 64.82 \]

\[ 100N = 324.1 \]

\[ \frac{100N}{100} = \frac{324.1}{100} \]

\[ N = 3.241 \]

**Answer:** 5% of 64.82 is 3.241
TO ESTIMATE WITH BASIC PERCENT PROBLEMS

100% represents the whole thing. Use this to start the estimate. The other % represents the other number, this is the part

\[
\begin{align*}
100\% & = \text{the value of the “whole”} \\
The \text{other } \% & = \text{the value of the “part”}
\end{align*}
\]

**Ex. A:** What is 5% of 64.82?

An estimate for Ex.A will look like this:

\[
\begin{align*}
100\% & = 64.82 \\
5\% & = N
\end{align*}
\]

For this estimate, it is helpful to know how to find 10% of any number. This is also a very valuable tool in life, especially when eating in a restaurant where you would need to leave a tip.

Ex. If the whole is 100% and you want to find 10% of the whole thing, you can make 100% look like 10% simply by moving the decimal point one place to the left: \(100\% = 10.0\% = 10\%\)

You can do the same thing on the number side:

In 64.82, moving the decimal point one place to the left we get 6.482

Now we know 6.482 is 10% of the whole thing, 64.82, but you only want 5% of 64.82

Getting back to the above estimate:

\[
\begin{align*}
100\% & = 64.82 \\
10\% & = 6.482 \\
5\% & = N \\
5\% & = 3.241
\end{align*}
\]

Isn’t 5% half of 10%? So divide both sides by 2 and get 5% = 3.241.

Notice, this is the same exact solution we found when solving the proportion.

**Answer:** 5% of 64.82 is 3.241

**NOTE:** Sometimes you can even solve the exact problem using the estimate!
Use proportions to solve the following problems.

1) What is $10\%$ of $579.5$?

2) $220\%$ of $30$ is what number?

3) $70$ is $80\%$ of what number?

4) $40.5\%$ of what number is $48.6$?

5) $195$ is what percent of $78$?

6) $372$ is what percent of $310$?
If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Decimal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Percent</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For questions 4 - 15, solve the following percent problems:

4) 16 is what percent of 40?  
5) What is $33\frac{1}{3}$% of 60?  
6) 0.5 is 20% of what number?
<table>
<thead>
<tr>
<th></th>
<th>Question</th>
<th></th>
<th>Question</th>
<th></th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>7)</td>
<td>32% of 200 is what number?</td>
<td>8)</td>
<td>16$\frac{2}{3}$% of what number is 0.7?</td>
<td>9)</td>
<td>What percent of 96 is 12?</td>
</tr>
<tr>
<td>10)</td>
<td>32$\frac{1}{2}$% of what number is 65?</td>
<td>11)</td>
<td>3 is what percent of 600?</td>
<td>12)</td>
<td>9 is 15% of what number?</td>
</tr>
<tr>
<td>13)</td>
<td>What is 58% of 45?</td>
<td>14)</td>
<td>What percent of 11 is 2.2?</td>
<td>15)</td>
<td>5.5% of 2 is what number?</td>
</tr>
</tbody>
</table>
Chapter 5.4

PERCENT WORD PROBLEMS

The easiest way to solve percent word problems is to set up proportions and solve for the unknown quantity. Be sure to answer the question that was asked. The easiest ratio to find in any problem is the percent ratio. Write the percent ratio and label it. Then write the number ratio and label it. It is essential to **write labels first** so the numbers are entered correctly.

“% of whole is part”

\[
P\% = \frac{P}{100} \quad \text{and} \quad \frac{P}{100} = \frac{\text{part}}{\text{whole}}
\]

**Ex. A:** 80% of BEP Math students that take the final are also eligible for the MAT 001 Exit Exam. If I had 32 students take the MAT 001 Exit Exam, how many students took the final?

Step 1: Numerical Information:
80% of BEP Math students take the Exit Exam.
80% is the percent number, write this ratio (with labels) first.
32 students took the MAT 001 Exit Exam
N = the total number of students who took the final

Step 2:

<table>
<thead>
<tr>
<th>Percent Ratio</th>
<th>Number Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{80}{100} \text{ take the MAT 001 Exit Exam} )</td>
<td>( \frac{32 \text{ students take the MAT 001 Exit Exam}}{N \text{ total students in the class}} )</td>
</tr>
</tbody>
</table>

\[
\frac{80}{100} = \frac{32}{N}
\]

80 (N) = 32 (100)

\[
\frac{80N}{80} = \frac{3200}{80}
\]

N = 40

Answer: 40 students took the final.

**NOTE:** Make sure the labels on the top are the same and the labels on the bottom are the same in both ratios. In other words, **MATCH LABELS!!!**
Problems:

1) If your monthly income is $3600 and you spend 25% of this on housing, how much do you spend on housing each month?

2) If you mix 12 pounds of sugar with water to get a water-sugar mixture that is 40% sugar, what is the total weight of the mixture?

3) If a basketball player makes 14 out of 16 free throws in a game, what is his or her shooting percentage?
4) If 160 people went to see a musical at the NCC Theater and 45% of the audience were children, how many adults went to see the musical?

5) If you mix 3 liters of pure acid with 9 liters of water, what is the % acid in this mixture?

6) If a student answered 36 questions correctly on the lab pre-test and received a score of 60%, how many questions were on the test?
Homework 5.4  Name:

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Percent</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) 66% of students going to any type of college require a remedial course in mathematics. If there are 22,000 students currently at NCC, how many of them needed to take a remedial course?

3) When a quarterback completes 25 out of 40 passes during a game, what is his pass completion percentage?
4) In a BEP 092 freshman class, 36 students complete their BEP math course and pass both the MAT 001 and MAT 002 exit exams. This is 5.76% of all BEP math students. How many students are enrolled in the BEP math course?

5) A student received a grade of 84% on a test and answered 21 questions correctly. How many questions did the student answer incorrectly?

6) If your net paycheck is $630 and this is 60% of your gross paycheck, how much is your gross paycheck?
7) If your net paycheck (your take-home pay) is $1383.55 a month and you spend 25% of your net income on rent, how much is your rent a month?

8) In the photography club, 40% of the 65 members own digital cameras. How many of the club members do not own a digital camera?

9) A union takes 1% of the workers’ gross salary for union dues. If someone pays $17.87 in union dues, what is their gross salary?
10) If you normally walk 4.2 miles per day and on Sunday you decide to keep going and walk 6.3 miles that day, what percent of your usual walk was Sunday’s walk?

11) Fred usually gives his mom $240 per month to help her out. In November, if he gave her 120% of what he normally gives her, how much money did he give her in November?

12) Your gross paycheck every other week is $1961.29. The social security deduction is $121.50, the medical deduction is $28.44, federal withholding tax deduction is $256.62, and the state withholding tax deduction is $93.35. What percent of your gross paycheck are all of your deductions combined? (Round the answer to the nearest whole percent.)
Chapter 5.5

SALES TAX PROBLEMS

“% of whole is part”

\[
P\% = \frac{P}{100} = \frac{\text{part}}{\text{whole}}
\]

Sales tax is a percentage of the purchase price of an item (before tax) and is added to the cost of the purchase.

\[
\text{% of the purchase price of the item before tax} = \text{$ sales tax}
\]

\[
\text{$ Sales Tax} = \text{Tax Rate (\%)} \times \text{$ Selling Price}
\]

<table>
<thead>
<tr>
<th>Percent Ratio</th>
<th>$ Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{P}{100})</td>
<td>(\text{tax}) (\div) (\text{selling price})</td>
</tr>
</tbody>
</table>

**Ex. A:** If the sales tax rate is 6% and the purchase price of the cell phone before tax is $675.00:

a) What is the tax?  
b) What is the total cost?

**Step 1: Numerical Information:**

- 6% = Sales tax rate
- $675 = the purchase price of the cell phone before tax
- N = Sales tax in dollars

**Step 2:**

\[
\text{$ Total Cost} = \text{$ Purchase price before tax} + \text{$ Sales tax}
\]

a) Find the sales tax:

\[
6 \% \text{ sales tax rate} = \frac{6}{100} \quad \frac{\text{sales tax}}{\text{total price before tax}}
\]

\[
\frac{6}{100} = \frac{N}{675}
\]

\[
100N = 6(675)
\]

\[
100N = 4050
\]

\[
N = 40.5 \text{ is the actual sales tax in dollars} \rightarrow \text{40.50}
\]

b) The total cost = cost of the cell phone before tax + amount of the tax

\[
$715.50 = \quad $675 + \quad $40.50
\]

**Answer:** The sales tax on the cell phone is 40.50.  
The total cost of the cell phone, including tax, is $715.50
Problems:

1) What is the sales tax on a TV selling for $515.50 (before tax) if the sales tax rate is 8%?

2) If tax on a jacket costing $110.00 is $7.70, ($117.70 = total cost), what is the tax rate?

3) If tax = $4.75 with a tax rate of 4%, a) What is the selling price? b) What is the total cost?
4) An Otter box costs $320. If the tax rate is 6.5%, a) What is the tax? b) What is the total cost?

5) A coat costs $257.20. If the tax on this coat is $12.86, what is the tax rate?

6) If tax on a pair of glasses is $9.45 and the tax rate is 4.5%, a) What is the selling price? b) What is the total cost?
Homework 5.5  Name:

If you are not familiar with any or all of the following terms or problems refer to your notes OR Glossary/Important Ideas and Concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Sales Tax</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Sales Tax Rate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3) Find a) the sales tax and b) the total cost of an iPad selling for $399 with a tax rate of $\frac{5}{8} \%$.

4) The sales tax on food in North Carolina is 6\%. How much did I spend on groceries (pre-tax) if the sales tax I paid was $10.58$?
5) The total cost for my metro card for the month was $134.40 and the sales tax was $14.40. Find the sales tax rate on the metro card.

6) A PC laptop costs $475 before tax. If the tax rate is 8% a) what is the sales tax and b) what is the total cost for the laptop?
Chapter 5.6

PERCENT INCREASE & DECREASE PROBLEMS

A percentage increase or decrease is based on an original amount before any increase/decrease occurs.

If you were to receive a raise in pay that is a percentage of your current salary, the raise is based on your current salary. This raise would be ADDED to your current salary to get the new salary.

If you were to buy an item on sale that is a percentage off the original price, the amount of the discount (decrease in price) is based on the original price (before the discount). This discount would be SUBTRACTED from the original price to get the sale price.

"% of whole is part" \[ P\% = \frac{P}{100} \]

\[ \frac{P}{100} = \frac{\text{part}}{\text{whole}} \]

% of \[ \left( \text{Original Amount} \right) \]

is \[ \left( \text{The amount of the increase} \right) \]

Original Amount + Increase = New Amount

% of \[ \left( \text{Original Amount} \right) \]

is \[ \left( \text{The amount of the decrease} \right) \]

Original Amount – Decrease = New Amount

Ex. A: Suppose your original salary is $43,600 and you will be getting a 7% raise in pay. What is the amount of the raise?

Numerical Information:

$43,600 = \text{original salary} \quad 7\% = \text{percent raise} \quad N = \text{dollar raise}

\[ \frac{7\% \text{ raise}}{100\% \text{ total}} = \frac{\$N \text{ raise}}{\$43,600 \text{ original}} \]

\[ \frac{7}{100} = \frac{N}{43,600} \]

\[ 100N = 7(43,600) \]

\[ \frac{100N}{100} = \frac{305,200}{100} \]

\[ N = 3052 \text{ is the actual raise in dollars} \to \$3052 \]

Answer: The amount of the raise is $3052.
Problems:

1) Suppose 3200 students take the placement test each semester. If we expect a 4.5% increase, how many more students will be taking the test? What will the new total be?

2) If gas prices recently increased from $2.09 to $2.29, what % increase is this (to the nearest tenth of a percent)?

3) If you buy clothes worth $250 and you have a coupon for “20% off your entire purchase,” how much will you pay for these clothes? (No tax week!)
4) If a necklace goes up in value from $1900 a few years ago to $2700 today, to the nearest 10\textsuperscript{th} of a percent, what is the percent increase?

5) If gas costs $2.29 today and there is an expected increase of 6\% in the next few months, what will the new price be after the expected increase happens?
Homework 5.6

Name: 

If you are not familiar with any or all of the following terms or problems refer to your notes OR glossary/important ideas or concepts OR go to lab.

<table>
<thead>
<tr>
<th>Term:</th>
<th>Official Definition:</th>
<th>In your own words and/or example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent Increase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent Decrease</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3) Your initial salary as a teacher is $35,600. After your first year you will receive an 8% raise. What will your raise be? What will your new salary be after your raise?

4) In 1997 there were 238 students in the BEP program. Today there are 663 students. To the nearest whole %, what percent increase has there been since the program started?
5) We have a 9.7% average increase per year in the BEP program. If we have 663 students this year, how many students can we expect to have in the program next year?

6) In recent years, gas prices have gone from $4.05 per gallon to $3.15 per gallon. What percent decrease is this? (Round your answer to the nearest whole percent.)

7) In 1952 a house was purchased for $13,000. It was sold this year for $465,000. What was the actual increase in value and what was the percent increase (round to nearest whole percent)?

8) A new car cost $19,362. One year after it was purchased its value had dropped to $14,857. How much did the car decrease in value in that first year of ownership? What was the percent decrease in value (round to nearest whole percent)?
9a) A pair of boots was purchased at a store where there was a big sale. They were giving a 20% discount on everything in the store. If the boots originally cost $175, what was the sale price (before tax)?

9b) Referring to the boots purchased in question 9a: At the register they gave you a coupon for an additional 10% off the purchase price. After the initial 20% discount and the additional 10% coupon, how much did you pay for the boots (before tax)?

9c) How much money did you save after both discounts were given?

9d) Based on the savings after both discounts, what was the actual % discount you received?
**Cross Products**

**Definition:** Cross products are the two products created when cross multiplying two fractions.

**Example:**

\[
\frac{4}{5} = \frac{8}{10} \\
4 \times 10 = 5 \times 8 \\
40 = 40
\]

40 and 40 are the cross products.

**Percent**

**Definition:** A percent is a special group of ratios with 100 in the denominator and any type of number (fraction, whole number, mixed number or decimal) in the numerator.

**Example:** Percents are written as N%.

The percent symbol is % and percent means “out of 100” or “over 100” or “divide by 100.”

\[
N\% = \frac{N}{100} = N \div 100
\]

**Proportion**

**Definition:** Two ratios form a proportion if their cross products are equal.

**Example:**

\[
\frac{4}{5} = \frac{8}{10} \\
4 \times 10 = 5 \times 8 \\
40 = 40
\]

Since 40 = 40 (the two ratios are equal), \(\frac{4}{5}\) and \(\frac{8}{10}\) are proportional.

**Sales Tax**

**Definition:** Sales Tax is the extra money added to the selling price of taxable items purchased.

**Example:** A used car costing $2000 before tax would have a sales tax of $152.50, bringing the total cost of the car to $2152.50, including tax.

**Sales Tax Rate**

**Definition:** The sales tax rate is a proportional amount based on the selling price of an item.

**Example:** If the sales tax rate is 8%, this means for every dollar you spend you pay an additional 8 cents in sales tax.

---

**Chapter 5 Important Ideas and Concepts**

**Cross Multiplying:** To cross multiply we multiply the numerator of one fraction by the denominator of the other fraction only across an equal sign. To cross multiply means to set the two cross products equal. See pages 216, 217, 253, 297, and 382.

**Percent Decrease/Increase** – Percent Decrease/Increase is the proportional amount by which an original number is decreased or increased. See page 421.
For questions 1 - 4, use the following information:

There are 23 animals in the animal shelter. Ten animals are dogs and 13 animals are cats.

<table>
<thead>
<tr>
<th>1) What is the ratio of dogs to animals in the shelter?</th>
<th>2) What is the ratio of cats to animals in the shelter?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) What is the ratio of cats to dogs?</td>
<td>4) What is the ratio of dogs to cats?</td>
</tr>
</tbody>
</table>

5) A recipe calls for 2 quarts of club soda for every 5 quarts of juice.
   a) What is the ratio of juice to club soda?
   b) Reduce this ratio to a unit ratio. (What is the ratio of juice to 1 quart of club soda?)

For questions 6 – 8, are the following ratios proportional? Explain why or why not:

| 6) \( \frac{3}{5} = \frac{9}{25} \) ? | 7) \( \frac{451}{1003} = \frac{3157}{7021} \) ? | 8) \( \frac{4}{\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{2}{6}} \) ? |

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For questions 1 – 12, complete the chart below:

<table>
<thead>
<tr>
<th>Fraction and/or mixed number</th>
<th>Decimal</th>
<th>Percent (mixed #/fraction/decimal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/16</td>
<td>1)</td>
<td>2)</td>
</tr>
<tr>
<td>3)</td>
<td></td>
<td>4)</td>
</tr>
<tr>
<td>0.(\overline{6})</td>
<td></td>
<td>70%</td>
</tr>
<tr>
<td>11(\frac{3}{4})</td>
<td>7)</td>
<td>8)</td>
</tr>
<tr>
<td>9)</td>
<td></td>
<td>10)</td>
</tr>
<tr>
<td>17.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11)</td>
<td>12)</td>
<td>145%</td>
</tr>
</tbody>
</table>
13) Of the 660 students enrolled in the BEP 092 math course 1/10 of them will stop coming for their own reasons, 3/20 of them will be dropped by the professors due to lateness, absences, or because they do not hand in their work, 1/5 will complete the course, but need to repeat it because they are not academically ready to move forward, 1/4 will complete the course and move on to MAT 001 and 3/10 of the students will not only complete the course satisfactorily but will go on to take MAT 002 (i.e., skip MAT 001).

<table>
<thead>
<tr>
<th>13a</th>
<th>What percent of students stop attending on their own?</th>
<th>13b</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>13b</th>
<th>What percent of students are dropped by their professors?</th>
</tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>13c</th>
<th>What percent of students complete the course, but need to repeat it?</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>13d</th>
<th>What percent of students pass and go onto MAT 001?</th>
</tr>
</thead>
<tbody>
<tr>
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<table>
<thead>
<tr>
<th>13e</th>
<th>What percent of students pass and go onto MAT 002?</th>
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</table>
For questions 1 - 12, solve the following percent problems:

<p>| | | | |</p>
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<thead>
<tr>
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<tbody>
<tr>
<td>1)</td>
<td>90 is what percent of 45</td>
<td>2)</td>
<td>What is $16 \frac{2}{3}$% of 12?</td>
</tr>
<tr>
<td>3)</td>
<td>4.2 is 6% of what number?</td>
<td>4)</td>
<td>37% of 89 is what number?</td>
</tr>
<tr>
<td>5)</td>
<td>85% of what number is 217.6?</td>
<td>6)</td>
<td>What percent of 35 is 21?</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
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<td>-----------------------------------------------</td>
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<tr>
<td>7) 40% of what number is 22?</td>
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<tr>
<td>8) 0.19 is what percent of 0.57?</td>
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<tr>
<td>9) 18 is 150% of what number?</td>
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<tr>
<td>10) What is 125% of 80?</td>
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<tr>
<td>11) What percent of 90 is 81?</td>
<td></td>
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<tr>
<td>12) 28% of 6.5 is what number?</td>
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<tr>
<td>1) 79% of students going to any community college are required to take a remedial algebra course in mathematics. If there are 22,000 students currently at NCC, how many of them need to take a remedial algebra course?</td>
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<tr>
<td>2) If a basketball player makes 5 out of 15 free throws during a game, what is his/her free throw success rate (percentage)?</td>
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<tr>
<td>3) Recently 257 students completed all of their BEP courses in one semester. This was 31% of the students registered for that particular semester. How many students were registered in BEP that semester?</td>
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<tr>
<td>4) A student received a grade of 64% on a test and answered 16 questions correctly. How many questions did the student answer incorrectly?</td>
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<tr>
<td>5) If your net paycheck (your take-home pay) is $1186.75 a month and you spend 32% of your net income on auto insurance, how much is your auto insurance a month?</td>
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<tr>
<td>6) Joe usually gives his sister $240 a month. He gave her $285 in October. What percent of what Joe normally gives her did he give her in October?</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1) Find a) the sales tax and b) the total cost of a generic tablet selling for $95 with a tax rate of $\frac{83}{4}$%.</td>
<td>2) The sales tax on food in Pennsylvania is 6%. How much did I spend on groceries (pre-tax) if I the sales tax I paid was $5.85?</td>
<td></td>
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<tr>
<td>3) The total cost for a daily latte purchase in one month is $98.69, including $7.84 tax. Find the sales tax rate. (Round to the nearest hundredth of a percent.)</td>
<td>4) The most expensive Kindle reader costs $269 before tax. If the tax rate is 8%, (a) what is the sales tax and (b) what is the total cost for this Kindle?</td>
<td></td>
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</tr>
<tr>
<td>5) What is the selling price of a phone if the tax rate is 7% and the tax on the phone is $13.93?</td>
<td>6) What was the tax rate for a used car that cost a total of $4121.06 if the selling price for the car was $3875? (Round to the nearest hundredth of a %.)</td>
<td></td>
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</tbody>
</table>
For questions 1-4, **use the fact that a pair of boots originally cost $149.99:**

1) If the boots (see above) were on sale for 20% off, what was the sale price (before tax)?

2) If you had a coupon for an additional 10% off your purchase, how much did you pay for the boots after the initial 20% discount and the 10% coupon (pre-tax)?

3) How much did you save after both discounts were given?

4) Calculate the amount the total savings (from question 3) represent as a single discount ("percent off") based on the original price before any coupons were applied. (Round to nearest whole %.)

5) If you start out earning $8 per hour and you receive a raise after your first year, which now has you earning $10.75 per hour, what % raise did you receive?

6) If the news reports say that gas prices have dropped by 85 cents a gallon in the last year, which was a 22% drop in prices, how much did gas cost last year?
5.1 extra practice answers:
1) 10 dogs/23 animals  
2) 13 cats/23 animals  
3) 13 cats/10 dogs  
4) 10 dogs/13 cats  
5a) 5 qts juice/2 qts club soda  
5b) 2.5 qts juice/1 qt club soda  
6) no*  
7) yes *  
8) yes *

5.2 extra practice answers:
1) 0.0625  
2) 6.25% = $\frac{6}{4}$% = $\frac{25}{4}$%  
3) 2/3  
4) $66\frac{2}{3}$% = $\frac{200}{3}$%  
5) 7/10  
6) 0.7  
7) 11.75  
8) 1175%  
9) $17\frac{7}{20} = \frac{347}{20}$  
10) 1735%  
11) $1\frac{9}{20} = \frac{29}{20}$  
12) 1.45  
13a) 10%  
13b) 15%  
13c) 20%  
13d) 25%  
13e) 30%

5.3 extra practice answers:
1) 200%  
2) 2  
3) 70  
4) 32.93  
5) 256  
6) 60%  
7) 55  
8) $33\frac{1}{3}$%  
9) 12  
10) 100  
11) 90%  
12) 1.82

5.4 extra practice answers:
1) 17,380 students  
2) $33\frac{1}{3}$%  
3) 829 students  
4) 9 questions *  
5) $379.76  
6) 118.75%

5.5 extra practice answers:
1) $8.31; \$103.31  
2) $97.50  
3) 8.63%  
4a) $21.52  
4b) $290.52  
5) $199  
6) 6.35%

5.6 extra practice answers:
1) $119.99  
2) $107.99  
3) $42  
4) 28%  
5) 34.375%  
6) $3.86

* These Questions & Answers may require some discussion.
NOTE: Every type of question does not appear in this review section. To see problems of every type covered in this course, please see Extra Practice/Chapter Review problems within each chapter.

Chapter 1

1) Write standard notation for eight trillion, two hundred twenty-five billion, five hundred million, four hundred eighty-six.

2) In the number 6,258,403, what do the following digits represent?
   a) 2          b) 4          c) 6          d) 5

3) Write expanded notation for 136,403.

4) Solve: $5692 + x = 7036$

5) Add or subtract, and simplify: a) 6 ft. 8 in.  b) 6 ft. 2 in.
   \[+ 5 \text{ ft.} \ 9 \text{ in.} \quad \text{and} \quad - 5 \text{ ft.} \ 4 \text{ in.}\]

6) Write a word name for: 12,032,000,105

7) Find the perimeter of a rectangle which measures 10 ft. by 4 ft.

8) The Smith’s well in Arizona is 6483 ft. deep. The Jones’ well in Colorado is 3976 ft. deep. How much deeper is the Smith’s well?

9) How many triangles are in the figure below? Make a table to show your work.

Chapter 2

10) One bottle of soda holds 12 fluid ounces. How many ounces are there in 24 bottles?

11) Find the product of 4312 and 648.

12) Solve for $x$: $12 \cdot x = 108$
13) The odometer reading on Joan’s car was 27,154 miles before leaving on her trip. When she returned, it read 27,518. If she purchased 14 gallons of gas, what was her gas mileage?

14) Find all the factors of 36.

15) Perform the indicated operations: $\sqrt{49} - 5 + 1$

16) Find the area of the shaded region:

```
<table>
<thead>
<tr>
<th>12 yd.</th>
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</thead>
<tbody>
<tr>
<td>4 yd.</td>
</tr>
<tr>
<td>3 yd.</td>
</tr>
<tr>
<td>9 yd.</td>
</tr>
</tbody>
</table>
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17) Match the example to the name of the law:

1) $7 + 2 = 2 + 7$  a) associative law
2) $3 + 0 = 3$  b) distributive law
3) $2(5 + 1) = (2 \times 5) + (2 \times 1)$  c) commutative law
4) $(1 \times 2) \times 3 = 1 \times (2 \times 3)$  d) identity for multiplication
5) $5 \cdot 1 = 5$  e) identity for addition

**Chapter 3**

18) Simplify: a) $\frac{9}{0}$  b) $\frac{0}{9}$  c) $\frac{9}{9}$  d) $\frac{9}{1}$

19) What fractional part is shaded? a)  b)

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20) Solve for x: a) $\frac{8}{12} = \frac{x}{3}$  b) $\frac{3}{5} = \frac{9}{x}$

21) Multiply $7 \times \frac{2}{3}$ and write your answer as a) a mixed #  b) as an improper fraction
22) Is $\frac{4}{5}$ equal to $\frac{5}{6}$? (YES or NO)

23a) Find the square of $\frac{9}{16}$

b) Find the square root of $\frac{9}{16}$

24) In an 8-slice pizza pie, $\frac{3}{4}$ have pepperoni.  a) How many slices have pepperoni?

b) How many slices do not have pepperoni?

25) On a map, 1 inch equals 27 miles. How many miles does $\frac{4}{3}$ inches represent?

26) If 1 inch represents 20 miles, how many inches represent 80 miles?

27) If $\frac{2}{11}$ of a number is 10, find the number.

28) A piece of wire $\frac{2}{3}$ m long is cut into 12 pieces. How long is each piece?

29) Change $5\frac{7}{8}$ to an improper fraction.

30) Multiply: $3\frac{1}{5} \times 3\frac{2}{3}$

31) A room is $6\frac{2}{3}$ yards long and $4\frac{1}{4}$ yards wide how much will it cost to carpet the room if carpet costs $16 \frac{1}{2}$ per sq. yd.?

Chapter 4

32) Give the formal word name for 147.053

33a) Write $\frac{81}{100}$ as a decimal.  B) Write $\frac{3}{5}$ as a decimal

34a) Round the nearest thousandth:  a) 41.84  b) 3.17

35) Estimate the product: 2.7 x 57.4

36) Sue bought 8 tacos that cost $1.29 each and tax was $0.85.

a) What was the total cost?

b) What would the change be from a $20 bill?
37) What is the cost of 12.5 lbs of cheese if it is sold for $3.95 per lb?

38) Complete using <, > or $\frac{1}{6}$ __ 0.16

Chapter 5

39) There are 5 boys to every 3 girls in the class. What is the ratio of boys to girls?

40a) Write 37.35% as a decimal.

41a) Write 0.07 as a percent.

42a) Write $\frac{13}{20}$ as a percent.

43a) 30 is 5% of what number?

44a) What number is 60% of 250?

45a) 75 is what % of 225?

46) James will get a 14% raise is salary next season. His present salary is $36,000.

   a) How much will his raise be?

   b) How much will James be earning after his raise goes into effect?

47) You and your friend graduate college. You land a job earning $34,000 a year and your friend is earning $36,500 a year. After the first year you get an $8000 raise and your friend gets a 12% raise. Who will earn a higher salary and how much more is this individual earning after the raises take effect?
1. 8,225,500,000,486

2a. 2 hundred thousands  b. 4 hundreds  c. 6 millions  d. 5 ten-thousands

3. 100,000 + 30,000 + 6,000 + 400 + 3

4. x = 1344

5a. 12 ft. 5 in.  b. 10 inches

6. Twelve billion, thirty two million, one hundred five

7. The perimeter is 28 ft.

8. The Smith well is 2507 ft. deeper.

9. The 3 – 1 bit triangles are B, C and D

   The 4 – 2 bit triangles are AB, AC, BD and CD

   There are no 3 bit triangles but there is the 1 – 4 bit triangle ABCD for a total of 8 triangles.

10. There are 288 oz. in 24 bottles.

11. 2, 794,176

12. x = 9

13. Her mileage was 26 mpg.

14. 1, 2, 3, 4, 6, 9, 12, 18, 36

15. 3

16. The area inside the shaded region is 96 sq. yds.

17. 1 - c, 2 - e, 3 - b, 4 - a, 5 - d

18a. undefined  b. 0  c. 1  d. 9

19a. 5/16  b. 7/4

20a. x = 2  b. x = 15

21. mixed # = \( \frac{2}{3} \)  b. improper fraction = \( \frac{14}{3} \)

22. No

23a. \( \frac{81}{256} \)  b. \( \frac{3}{4} \)

24a. 6  b. 2
25. \( \frac{4}{3} \) represents 36 miles.

26. 4 inches would represent 80 miles.

27. The number is 55.

28. The length of each piece is \( \frac{1}{18} \) m.

29. \( \frac{47}{8} \)

30. \( 11 \frac{11}{15} \)

31. \( 467 \frac{1}{2} \) is the cost to carpet the room.

32. one hundred forty-seven and fifty-three thousandths

33a. 0.81    b. 0.6

34a. 41.848    b. 3.172

35. 180

36a. $11.17    b. $8.83

37. The cheese costs $49.38

38. >

39. 5 boys to 3 girls = 5 boys : 3 girls = \( \frac{5 \text{ boys}}{3 \text{ girls}} \)

40a. 0.3735    b. \( \frac{3}{2} \)

41a. 7%    b. 240%

42a. 65 %    b. 12%

43a. 600    b. 50

44a. 150    b. 24.5

45a. 33 \( \frac{3}{3} \)% or 33 \( \frac{1}{3} \)%    b. 20%

46a. His raise will be $5040    b. his new salary will be $41,040

47. You will be earning $42,000 and your friend will be earning $40,880. Your salary is now higher by $1,120.
Chapter 1 HW – answers

* These Questions & Answers may require some discussion.

1.1 answers:


19) 50,000,000 + 2,000,000 + 10,000 + 2000 + 800 + 3
20) 10,000,000,000 + 3,000,000,000 + 6000 + 20 + 5
21) 700,000,000,000,000 + 2,000,000,000,000 + 30,000,000,000 + 400,000 + 50,000
22) 14,005,927 23) 86,000,013,040 24) 5,000,802,004,007

1.2 answers:

5) 12,345,678  6) 1,234,567,891,234  7) 12,345,678,912,345

8) Two hundred one million, twelve thousand, one hundred eighty five
9) Seventy billion, eighty three million, nine
10) Forty six trillion, eight hundred thirty two million, twelve thousand
11) 312,186,529,716 12) 8,012,007,000 13) 7,000,000,017,091 14) hundreds
15) ten-thousands 16) (one) billions 17) 0 18) 3 19) 6 20) 5

1.3 answers:

6) 17  7) 0  8) *  9) 190 10) 23 11) 1488 12) 7 years 3 months
13) 6 ft 8 in 14) 9 hrs 10 min 15) 14 lbs 4 oz 16) 8 triangles (see below for details)
17) 10 triangles (see below for details) 18) 10 matches Challenge) 45 handshakes

16)

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th># of $\triangle$ s</th>
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<tbody>
<tr>
<td>1 – bit</td>
<td>$b$ $c$ $d$</td>
<td>3</td>
</tr>
<tr>
<td>2 – bit</td>
<td>$ab$ $ac$ $bd$ $cd$</td>
<td>4</td>
</tr>
<tr>
<td>3 – bit</td>
<td>$abcd$</td>
<td>0</td>
</tr>
<tr>
<td>4 – bit</td>
<td>$abcd$</td>
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</table>

Total: 8 triangles
### 1.3 Answers (cont.):

17)  

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<th># of △s</th>
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<tbody>
<tr>
<td>1 - bit</td>
<td>a b c d</td>
<td>4</td>
</tr>
<tr>
<td>2 - bit</td>
<td>ab bc cd</td>
<td>3</td>
</tr>
<tr>
<td>3 - bit</td>
<td>abc bcd</td>
<td>2</td>
</tr>
<tr>
<td>4 - bit</td>
<td>abcd</td>
<td>1</td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td>10 triangles</td>
</tr>
</tbody>
</table>

### 1.4 Answers:

5a) 4390  
5b) 4400  
5c) 4000  
6a) 900  
6b) 900  
6c) 1000  
7a) 72,880  
7b) 72,900  
7c) 73,000  
8a) 970  
8b) 970  
8c) 3500  
9a) 4390  
9b) 4400  
9c) 4000  
10a) 900  
10b) 900  
10c) 1000  
11a & b) *  
12a) $50,000; $50,000  
12b) *

### 1.5 Answers:

8a) *  
8b) no *  
9a) yes *  
9b) no *  
10a) no*  
10b) no*  
11) x = 26  
12) x = 52  
13) x = 176  
14) 16 feet 11 inches  
15) 9 weeks 3 days  
16) 6 hrs 24 min  
17) hundreds = 1; tens = 7; ones = 5  
18) hundreds = 4; tens = 4; ones = 7  
19) thousands = 5; hundreds = 0; tens = 5; ones = 8

### 1.6 Answers:

4) 246  
5) 378  
6) 2816  
7) 20  
8) 300  
9) *  
10) *  
11) $751 more  
12) 7 weeks 1 day  
13) 6 ft 9 in  
14) 1 hr 44 min  
15) tens = 2; ones = 3  
16) hundreds = 8; tens = 0; ones = 1  
17) ten-thousands = 2; thousands = 2; hundreds = 0; tens = 5; ones = 0

### 1.7 Answers:

1) 112 students  
2) 14 hours  
3) $818  
4) 1 hour 32 minutes  
5) 36 inches or 3 feet  
6) My aunt was 7 inches taller  
7) 11:13am  
8) 237 pages  
9) 7 years 8 months
Chapter 2 HW – answers

* These Questions & Answers may require some discussion.

2.1 answers:

7) You get the number itself
8) One and the number itself

9) 42 and 13 are the factors, 546 is the product
   NOTE: Because of this, 546 is a multiple of 42 and 546 is a multiple of 13.

10) 71 and 6 are the factors, 426 is the product.
   NOTE: Because of this, 426 is a multiple of 71 and 426 is a multiple of 6.

11) 1, 3, 5, 15
12) 1, 2, 3, 6, 9, 18
13) 1, 5, 25

14) Commutative Law
15) Commutative Law
16) (6 x 7) x 8

2.2 answers:

6) *
7) 5074
8) 81,403

9) 2,204,202
10) 800,000
11) 200,000

12) 35,000
13) 5,600,000
14) 300,000

15) 1,200,000
16) 2,620,800 bottles

2.3 answers:

2) \( x = 7 \) ft
3) *
4) *
5) *

6) 70 sq ft
7) 70 sq ft
8) 138 sq in
9) 98 sq yds

2.4 answers:

8) \( x = 9 \)
9) \( x = 7 \)
10) \( x = 441 \)
11) \( x = 3000 \)

12) no *
13a) 0
b) undefined
c) 1
d) 51

14) no *
15) no *

2.5 answers:

8) 203
9) 604
10) 659 r 2
11) 27,180 r 1
12) 215 R34
13) 312

14) 13
15) 593
16) 125 full bags with 10 extra marbles *
17) 62 *
2.6 answers:

5) Area = 540 sq ft; Perimeter = 96 ft
6) $786
7) no 8) yes 9) $480 10) $162 11) $642 12) $1250
13) 8 miles per gallon 14) $105 15) 31 miles per gallon (mpg)
16a) 52 paychecks 16b) $1248 17a) 24 paychecks 17b) $2704
18a) 26 paychecks 18b) $2496 Challenge problems: 1) $886 2) $300*

2.7 answers:

4) * 5) * 6) 700 and 350 7) all of them

2.8 answers:

6) no * 7a) 2 7b) * 8a) prime 8b) composite
8c) prime 8d) composite 8e) composite 8f) neither
9) 186 = 2 x 3 x 31 9) 255 = 3 x 5 x 17 10) 1560 = 2 x 2 x 2 x 3 x 5 x 13
11) ** 6438 = 2 x 3 x 29 x 37

2.9 answers:

7) no * 8) 4 x 4 x 4 9) 67 x 67 x 67
10) 2 x 2 x 3 x 3 x 4 x 4 x 4 11) 3 x 5 x 11 x 11 x 17 x 17 x 17
12) 7⁶ 13) 81⁴ 14) 4³ x 8² 15) 2³ x 10²
16) 4096 17) 300 18) 1372 = 2² x 7³ 19) 9000 = 2³ x 3² x 5³
20) 625 21) 5 22) 4 23) 256
24) 50,625 25) 15 26) 13 27) 28,561

2.10 answers:

8) 8 9) 64 10) 1 11) 3
12) 10 13) 89 14) 22 15) 9
Challenge problems: 16) 16 17) 1
Chapter 3 HW – answers

3.1 answers:
6) 7/8  7) 6/8 = ¾  8) 7/4 = 1 ¾  9) 1/2  10) 3/4  
11)  12) ½ the pie remains  13) 11/8 of a pie remains  
14) 2 1/3  15) 7 2/3  16) 5 18/31  17) 3/4  18) 5 3/4  19) 9 1/3  
20) 3/3  21) 68/13  22) 499/31  23) 13/4  24) 44/13  25) 45/7  

3.2 answers:
14) not equal  15) 7/9  16) 5/3  17) 1/5  18) 3/2  19) 13/22  

Challenge problems with algebra:
1) \( \frac{1}{x^3} \)  2) \( \frac{x^3}{2} \)  3) \( \frac{2y^3}{7x} \)  4) \( \frac{4}{7y^3} \)  5) \( \frac{3y^2}{2xz^2} \)  

3.3 answers:
7)  8)  9) ¾ x ½  10) 6/55  11) 3/17  12) 5/18  
13) 1/18  14) 6  15) 4  16) 5/9  17) 1\frac{1}{3}  18) 4/7  
19) 256/625  20) 121/49  21) 12\frac{1}{4}  22) 4/5  23) 11/7  24) 1\frac{1}{4}  
25) 16/81  26) 2/3  27) 3\frac{13}{81}  28) 2401/6561  29) 7/9  30) 1\frac{1}{3}  

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* These Questions & Answers may require some discussion.

3.4 answers:

3) * 4) 1/8 5) 3 6) 1 7) 2/9 8) * 9) * 10) 1/5
11) 5 12) 2/3 13) 3/2 14) $2 \frac{2}{3}$ 15) 3/8 16) 1 1/2 17) 1 $\frac{11}{16}$

3.5 answers:

10) N = 18 11) N = $5\frac{1}{3}$ 12) 2 13) 1/8 14) 1/9 15) 49
16) 4 17) 18 18) 5/13 19) 1/6

3.6 answers:

1) 100 2) 16 3) 9 miles 4) 4000 students 5) $90,000$ 6) $10$
7) $1/16$ yard 8) 16 pieces 9) 9 gallons 10) 16 gallons 11) 375 calories 12) $18$ per hr
13) 48 students 14) 147 students 15) 2/3 yards 16) $2\frac{1}{3}$ minutes
17) $4\frac{1}{2}$ pieces or 4 pieces with some extra wood 18) $50\frac{3}{4}$ 19) 16 bags 20) $78$

Challenge Problems: 21a) 8/15 hour 21b) 32 minutes 22) $816$
Chapter 4 HW – answers

4.1 answers:

6) Hundredths  7) Millionths  8) Thousands

9) Fourteen and seven hundredths

10) Fourteen and seven hundred thousandths

11) One hundred forty thousand, seven millionths

12) Twelve and three hundred twelve thousandths

13) One thousand, two hundred thirty-one and two tenths

14) One hundred twenty-three and twelve hundredths

15) 3  16) 4  17) 6  18) 0.15  19) 2005.101

20) 87.00953  21) 15.0011  22) 8.0108  23) 0.016008

4.2 answers:

6) \(\frac{3}{4}\)  7) \(\frac{17}{200}\)  8) \(\frac{8}{1}\)  9) \(\frac{125}{4}\)  10) \(6\frac{37}{5000}\)  11) \(\frac{5}{16}\)

12) \(\frac{8}{25}\)  13) \(\frac{9}{1250}\)  14) \(\frac{141}{2000}\)  15) \(\frac{71}{1000}\)  16) \(11\frac{219}{1000}\)  17) \(\frac{3}{50,000}\)

18) \(\frac{1}{4}\)  19) \(\frac{1}{2}\)  20) \(\frac{1}{8}\)  21) \(\frac{7}{8}\)  22) \(\frac{1}{9}\)  23) \(\frac{1}{3}\)

24) \(\frac{5}{8}\)  25) \(\frac{1}{10}\)  26) \(\frac{3}{4}\)  27) \(\frac{2}{3}\)  28) \(\frac{1}{5}\)  29) \(\frac{3}{8}\)

30) \(3.\overline{6}\)  31) 2.5  32) 2.75  33) 3.\(\overline{3}\)  34) 2.25  35) 0.2

36) 0.125  37) 0.\(\overline{1}\)  38) 0.375  39) 0.875  40) 1.1  41) 0.625
* These Questions & Answers may require some discussion.

### 4.3 answers:

4a) 87.04  
    b) 87.036  
    c) 87.0  
    d) 87  
    e) 90  

5a) 4.96  
    b) 4.964  
    c) 5.0  
    d) 5  
    e) 0  

6a) 11.09  
    b) 11.091  
    c) 11.1  
    d) 11  
    e) 10  

7a) 2051.90  
    b) 2051.896  
    c) 2051.9  
    d) 2052  
    e) 2050  

8a) 82.83  
    b) 82.833  
    c) 82.8  
    d) 83  
    e) 80  

9) $14  
10) $100,000

### 4.4 answers:

1) \(x = 17.24\) inches; \(y = 4.42\) inches  
2) \(P = 51.02\) inches  
3) \(A = 116.59\) square inches  
4) 17.921  
5) 9.321  
6) 58.570  
7) 3.168  

8) $315.18  
9) $1539.99 (over time pay = $35.27 per hr.)  
10) $14.11  
11) 29.2 mpg  
12) $8.76  
13a) $124.38  
13b) $15.55 (*tip *)

### 4.5 answers:

6) 0.45  
7) 2  
8) 1.64  
9) 1.79  

10) 4  
11) 10  
12) 1  
13) 7.96  

14) 0.0612; 0.612; 2.016; 2.162; 6.12; 6.2  
15) 0.126; 0.26; 0.42; 4.0026; 4.26  

16) 0.08; 1/8; 1/4; 0.34; 0.4; 3/4  
17) 0.7; 3/4; 4/5; 0.9; 7.5  

18) 12.0010324 < 12.0100324  
19) 0.1205 > 0.005006  

20) 1/9 > 0.1  
20) 1/27 < 0.37  

22) The 12 ounce box  
23) * The 3-pack is the better buy
Chapter 5 HW – answers

* These Questions & Answers may require some discussion.

5.1 answers:

5) 9/17  6) 8/17  7a) 9 female to 8 male  7b) 9 female:8 male  7c) 9 female/8 male
8a) 8 male to 9 female  8b) 8 male:9 female  8c) 8 male/9 female
9a) 8 cups/2 loaves  9b) 4 cups /1 loaf  10) no*  11) yes*  12) yes*

5.2 answers:

4) 0.625  5) 62.5%  6) 42 1/2  7) 4250%  8) 2/5  9) 0.4
10) 2.2  11) 220%  12) 1/3  13) 33 1/3%  14) 33/200  15) 0.165
16a) 25%  16b) 12.5%  16c) 12.5%  16d) 50%

5.3 answers:

4) 40%  5) 20  6) 2.5  7) 64  8) 4.2  9) 12.5%
10) 200  11) 0.5%  12) 60  13) 26.1  14) 20%  15) 0.11

5.4 answers:

2) 14,520 students  3) 62.5%  4) 625 students  5) 4 incorrect answers  6) $1050
7) $345.89  8) 39 people  9) $1787  10) 150%  11) $288  12) 25%

5.5 answers:

3a) $34.41  3b) $433.41  4) $176.33  5) 12%  6a) $38  6b) $513

5.6 answers:

3) $2848; $38,448  4) 179%  5) 727 students  6) 22%  7) $452,000; 3477%
8) $4505; 23%  9a) $140  9b) $126  9c) $49  9d) 28%
Glossary

Area
Definition: Area measures the size of a flat surface using square units. The formula for the area of a rectangle is Area = Length x Width, meaning:
Total # of squares = # of squares per row x # of rows
Example:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

4 squares/row x 3 rows = 12 squares
This rectangle is 12 square units

Associative Law of Addition
Definition: The associative law of addition states that regrouping more than 2 numbers results in identical sums. (Number order remains the same.)
Example: (1 + 2) + 3 = 1 + (2 + 3), regrouping numbers will not change the final sum.

Associative Law of Multiplication
Definition: The associative law of multiplication states that regrouping more than 2 numbers results in identical products. (Number order remains the same.)
Example: (1 x 2) x 3 = 1 x (2 x 3), regrouping numbers will not change the final product.

Base
Definition: A base is the number being repeatedly multiplied in an exponential expression.
Example: In the exponential expression, \(5^4\), the base is 5, the repeated factor.

Commutative Law of Addition
Definition: The commutative law of addition states that the sum of 2 numbers is the same regardless of number order.
Example: 3 + 5 = 5 + 3, switching (commuting) the order of the numbers doesn't change the sum.

Commutative Law of Multiplication
Definition: The commutative law of multiplication states that the product of 2 numbers is the same regardless of number order.
Example: 3 x 5 = 5 x 3, switching (commuting) the order of the numbers doesn't change the product.

Composite Number
Definition: A composite number is a number that has 3 or more factors.
Example: The factors of 9 are 1, 3, and 9, so 9 is a composite number because it has at least 3 factors.

Counting Numbers – See Natural Numbers

Cross Products
Definition: Cross products are the two products created when cross multiplying two fractions.
Example:

\[
\begin{array}{c}
4 \\
5 \\
\end{array} = \begin{array}{c}
8 \\
10 \\
\end{array}
\]

\(4 \times 10 = 5 \times 8\)
40 = 40
40 and 40 are the cross products.

Cube
Definition: A cube of a number is the number times itself three times. Write the number as a factor 3 times.
Example: The cube of 4 is \(4^3 = 4 \times 4 \times 4 = 64\)

Decimal-Fraction
Definition: A decimal-fraction is a fraction with the denominator equal to a power of ten.
Example:

\[
\begin{array}{c}
\text{1}\text{0} \\
\text{1}\text{00} \\
\text{1}\text{000} \\
\text{10,000} \\
\end{array}, \text{Etc.}
\]

Denominator
Definition: A denominator is the bottom number in a fraction. It is the total number of equal size portions you break one whole thing into.
Example: If the denominator is 4, break the whole thing into 4 equal size portions.

\[
\begin{array}{c}
1 \\
4 \\
\end{array}
\begin{array}{c}
1 \\
4 \\
\end{array}
\begin{array}{c}
1 \\
4 \\
\end{array}
\begin{array}{c}
1 \\
4 \\
\end{array}
\]

Difference
Definition: 1) A difference is the answer to a subtraction problem.
Example: In the sentence 10 - 6 = 4, 4 is the difference between 10 and 6.
Definition: 2) A difference is the distance between two numbers on the number line.
Example: There are 4 spaces between 6 and 10 on the number line.
Digits
Definition: The first ten whole numbers.
Example: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are the digits.

Distributive Law
Definition: The distributive law states that adding two numbers first then multiplying by a factor is the same as multiplying each number by the factor and then adding those products.
Example: 
\[12 \times (2 + 4) = (12 \times 2) + (12 \times 4)\]
Check: \(12 \times 6 = 24 + 48\)
\[72 = 72\]

Divisor
Definition: A divisor is the number that is being repeatedly subtracted from an original number to determine how many complete groups are contained in the original number.
Example: In the division sentence:
\[12 \div 3 = 4\]
3 is the divisor and there are 4 full groups of 3 contained in the number 12.

Equal to (=)/Equation
Definition: An equation states that two expressions are equal. Two expressions are equal if they have the same value.
Example: \(8 = x + 5\) is an equation. It states that 8 has the same value as \(x+5\)

Expanded Notation
Definition: The sum of the values of the NON-ZERO digits in a number.
Example: \(2,000,305,076\) in expanded notation is \(2,000,000,000+300,000+5,000+70+6\).

Exponents (Powers)
Definition: An exponent (power) is the number in an exponential expression that tells you how many times to repeatedly multiply the base by itself (or how many times to write the base as a factor).
Example: In the exponential expression \(5^4\), the exponent is 4, so the factor 5 is multiplied by itself 4 times (5 is written as a factor 4 times).
\[5 \times 5 \times 5 \times 5 = 625\]

Factor
Definition: Factors are the numbers being multiplied.
Example: In the multiplication sentence: \(12 \times 3 = 36\), 12 and 3 are two factors of 36.

Fraction
Definition: A fraction is any number expressed as a numerator over a denominator.
Example:
\[
\frac{N}{D} = \begin{array}{c}
\text{Any Whole Number} \\
\text{Any Natural Number}
\end{array}
\]
OR
\[
\frac{N}{D} = \begin{array}{c}
\text{Any Whole Number} \\
\text{Any Whole Number, except zero}
\end{array}
\]

Greater than (>)
Definition: A number is greater than another number if it is located to the right of the other number on the number line.
Example: \(15 > 2\) is read as “15 is greater than 2”
NOTE: 15 is located to the right of 2 on the number line.

Group Names
Definition: Numbers are arranged in 3-digit groups and each group is named.
Example: In 200,736 thousands is the group name for the 200 and there is no stated group name for the 736 (its group name is units, but is not stated).
Example: Trillions, Billions, Millions, Thousands, Units, these are the first five group names.

Identity Property of Addition
Definition: The identity property of addition states that adding zero to a number gives the same number as the sum.
Example: \(18 + 0 = 18\), (18 keeps its “identity”).

Identity Property of Multiplication
Definition: The identity property of multiplication states that multiplying a number by 1 gives the same number as the product.
Example: \(18 \times 1 = 18\), (18 keeps its “identity”).

Improper Fraction
Definition: An improper fraction is a fraction where the numerator is greater than or equal to the denominator.
Example: \(\frac{5}{3}, \frac{13}{13}, \frac{12}{8}\) are all examples of improper fractions.

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Less Than ( < )
**Definition:** A number is less than another number if it is located to the left of the other number on the number line.
**Example:** 4 < 9 is read as “4 is less than 9”
**NOTE:** 4 is located to the left of 9 on the number line.

Mixed Number
**Definition:** A mixed number is a whole number combined with a proper fraction.
**Example:** $3 \frac{4}{9}$, $7 \frac{1}{2}$, $13 \frac{12}{15}$ are all mixed numbers.

Natural or Counting Numbers
**Definition:** Natural or Counting Numbers are the numbers that start with 1 and continue forever.
**Example:** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11… are the Natural Numbers.

Numerator
**Definition:** A numerator is the top number in a fraction which represents the number of equal size portions you want.
**Example:** If the numerator is 3 then you want three portions. (In other words, shade three portions.)

$\frac{3}{4}$ is shaded:

<table>
<thead>
<tr>
<th></th>
<th>1/4</th>
<th>1/4</th>
<th>1/4</th>
<th>1/4</th>
</tr>
</thead>
</table>

Percent
**Definition:** A percent is a special group of ratios with 100 in the denominator and any type of number (fraction, whole number, mixed number or decimal) in the numerator.
**Example:** Percents are written as N%.
The percent symbol is % and percent means “out of 100” or “over 100” or “divide by 100.”

$$N\% = \frac{N}{100} = N \div 100$$

Perfect Square
**Definition:** A perfect square is a result of multiplying any whole number by itself.
**Example:** $0^2 = 0$, $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$…
The perfect squares are 0, 1, 4, 9, 16…

Perimeter (sum of all sides)
**Definition:** The sum of the lengths of all the sides of any shape.
**Example:** A rectangle whose length is 5 inches and width is 3 inches has a perimeter of $5 + 3 + 5 + 3 = 16$ inches.

Powers
**Definition:** (See exponents).

Prime Factorization
**Definition:** A prime factorization is the writing of a composite number as a product of its prime factors.
**Example:** 12 can be written as the product of the prime factors 2 and 3.
$12 = 2 \times 2 \times 3$ (Think of “Factor Trees”).

Prime Factorization with Exponents
**Definition:** A prime factorization that uses exponents for repeated prime factors instead of writing them separately.
**Example:** $12 = 2^2 \times 3$

Prime Number
**Definition:** A prime number has exactly two different factors, 1 and itself.
**Example:** The factors of 2 are 1 and 2 so 2 is a prime number.
The first ten prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

Product
**Definition:** A product is the answer to a multiplication problem.
**Example:** In the multiplication sentence $4 \times 5 = 20$, the product is 20.

Proper Fraction
**Definition:** A proper fraction is a fraction where the numerator is smaller than the denominator.
**Definition:** A proper fraction represents a piece or a portion of a whole thing.
**Example:** $\frac{2}{5}$, $\frac{5}{11}$, $\frac{4}{8}$ are examples of proper fractions.
Proportion
Definition: Two ratios form a proportion if their cross products are equal.
Example:
\[
\frac{4}{5} = \frac{8}{10}
\]
\[
4 \times 10 = 5 \times 8
\]
Since 40 = 40 (the two ratios are equal), \(\frac{4}{5}\) and \(\frac{8}{10}\) are proportional.

Remember to include labels when labels are given.

Quotient
Definition: A quotient is the answer to a division problem.
Example: In the division sentence, \(20 \div 5 = 4\), the quotient is 4.

Ratio
Definition: A ratio is a comparison of two numbers and can be written three different ways.
Example: The ratio that compares 3 to 7 can be written as follows:

Word Form: 3 to 7
Colon Form: 3:7
Fraction Form: \(\frac{3}{7}\)

Remember to include labels when labels are given.

Remainder
Definition: A remainder is whatever is left after you have subtracted the divisor as many times as possible from a given quantity.
Example: In the division sentence, \(22 \div 5 = 4r2\), because \(22 - 5 - 5 - 5 - 5 = 2\), the quotient is 4 and the remainder is 2.

OR
Definition: A remainder is the leftover quantity in a division problem.
Example: In the following division sentence \(23 \div 5 = 4 r 3\), the remainder is 3.

Sales Tax
Definition: Sales Tax is the extra money added to the selling price of taxable items purchased.
Example: A used car costing $2000 before tax would have a sales tax of $152.50, bringing the total cost of the car to $2152.50, including tax.

Sales Tax Rate
Definition: The sales tax rate is a proportional amount based on the selling price of an item.
Example: If the sales tax rate is 8%, this means for every dollar you spend you pay an additional 8 cents in sales tax.

Square
Definition: A square of a number is the number squared, that is, the number times itself (write the number as a factor twice).
Example: The Square of \(36 = 36^2 = 1296\).
Remember: \(36^2 = 36 \times 36\)

Square Root
Definition: The square root of a given number asks you to determine what number must be multiplied by itself to get the number under the radical sign.
Example: The Square Root of \(36 = \sqrt{36} = 6\).
Remember: \(\sqrt{36} = \sqrt{6 \times 6} = 6\)

Standard Notation
Definition: Standard notation is when a number is written using digits.
Example: 2,000,305,076 is a number written in standard notation.

Sum
Definition: 1) Sum as a verb is to add two or more numbers.
Example: Sum 4 and 5 means “add 4 and 5”
Definition: 2) Sum as a noun is the answer to an addition problem.
Example: In the addition sentence, \(4 + 5 = 9\), 9 is the sum of 4 and 5.

Whole Numbers
Definition: Whole numbers are the natural numbers with zero added at the beginning of the list.
Example: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, …are the Whole Numbers.

Word Names
Definition: A word name for a number is the number written using words.
Example: A word name for 2,000,305,076 is two billion, three hundred five thousand, seventy-six.
**Better Buy:** Any item identical in quantity and quality to another, which costs less, is a Better Buy.  
*See pages 149 and 356.*

**Check for Long Division:** \[ \text{Quotient} \times \text{Divisor} + \text{Remainder} = \text{Dividend} \]
\[ \text{Top Number} \times \text{Outside Number} + \text{Remainder} = \text{Inside Number} \]
*See page 135.*

**Comparing Decimals:** Line up the decimal points in the numbers and going from left to right find the first column where the digits are different and start your comparison there.  
*See pages 355 - 356.*

**Converting Improper Fractions to Mixed Numbers:** Convert an Improper Fraction to a Mixed Number by Dividing the Numerator by the Denominator. The Quotient is the Whole Number and the Remainder is the Numerator of the Proper Fraction. Keep the Denominator, it does not change.  
*See page 209.*

**Converting Mixed Numbers to Improper Fractions:** Convert a mixed number to an improper fraction by Multiplying the Denominator by the Whole Number then Add the Numerator of the Proper Fraction to create the new Numerator for the Improper Fraction. Keep the Denominator, it does not change.  
*See page 208.*

**Counting Triangles:** Determining the total number of triangles in a given shape that can be made from the individual pieces and all possible combinations of the pieces that have common sides.  
*See pages 29 - 33.*

**Cross Multiplying:** To cross multiply we multiply the numerator of one fraction by the denominator of the other fraction only across an equal sign.  
*Example:* \[ \frac{4}{5} = \frac{8}{10} \]
\[4 \times 10 = 5 \times 8 \]
\[40 = 40\]
*See pages 216, 217, 253, 382.*

**Cutting/Separating into Equal Size Pieces:** When cutting or separating something into equal size pieces or portions, it is being “evenly divided.” Start with the thing that is being cut or separated followed by the division sign followed by the divisor which is either the number of equal portions or the size of each of portion.

\[(\text{total amount}) \div (\text{amount in each portion}) = (\text{total number of portions})\]
\[\text{Or}\]
\[(\text{total amount}) \div (\text{total number of portions}) = (\text{amount in each portion})\]
*See page 124-126, 184, 186, 275, 276.*

**Division Notations:** There are three different notations for division. In other words, there are three different ways to write a division sentence. *For Example:* fifty-six divided by eight equals seven.

1) \[ 56 \div 8 = 7 \]
2) \[ \frac{56}{8} = 7 \]
3) \[ 8)56 \]
\[ \text{Left} \div \text{Right} \quad \text{Top} \div \text{Bottom} \quad \text{Inside} \div \text{Outside or Outside “goes into” Inside} \]
*See page 123, 185, 186.*
Division Rules/Facts:

1) Any Number Divided by One is that Number
   \[ \frac{N}{1} = N \]
   \[ \frac{N}{1} = N \]
   \[ \frac{N}{1} = N \]

2) Zero divided by Anything is Zero
   \[ 0 \div N = 0 \]
   \[ 0 \]
   \[ \frac{0}{N} = 0 \]
   \[ N \div 0 = 0 \]
   \[ \text{Check: } 0 \times N = 0 \]
   \[ \text{Check: } 0 \times N = 0 \]
   \[ \text{Check: } 0 \times N = 0 \]

3) Anything divided by Zero is UNDEFINED
   \[ N \div 0 = \text{Undefined} \]
   \[ N \]
   \[ \frac{N}{0} = \text{Undefined} \]
   \[ 0 \]
   \[ \text{Check: } 0 \times \text{something} = N \]
   \[ \text{which is impossible!} \]
   \[ \text{That is why Division by Zero is UNDEFINED!} \]

See page 128, 185, 215.

Estimate: To find an approximate answer to any mathematical problem by first rounding all numbers (using the highest place value) then performing the indicated operation with the rounded numbers.

See pages 39, 42, 104, 340.

Fraction/Decimal Conversion

All fractions can be written in decimal form by dividing the numerator by the denominator. All decimals can be changed to fractions by naming the decimal and writing in fraction (mixed number) form.

REMEMBER TO REDUCE THE FRACTION TO LOWEST TERMS!

Example:

\[ \frac{3}{4} = 3 \div 4 = 0.75 \]

\[ 0.75 = \text{seventy-five hundredths} = \frac{75}{100} = \frac{3 \times \frac{25}{4}}{2 \times 2 \times \frac{25}{4}} = \frac{3}{4} \]

so, \[ \frac{3}{4} = 0.75 \] and \[ 0.75 = \frac{3}{4} \]

See pages 325, 327, 330, 331.

Fraction Division: In fraction division you must change the operation to multiplication by the reciprocal of the divisor.

Memory Trick: “Keep Times Flip” i.e. Keep the first fraction the same, change \( \div \) to \( \times \), and flip the second fraction.

See pages 239, 297.
**Fraction Multiplication:** A product of fractions is the result of the product of the numerators over the product of the denominators. All fraction answers must be written in lowest terms. Because of this, it’s easier to reduce first with Algebra before the multiplication.  
**Remember:** Stretch then Factor, Cancel, and Multiply.  
*See pages 228, 297.*

**Missing Digit Addition & Subtraction:** Determine the digits necessary to make a mathematical problem correct.  
*See pages 47 – 48, 60, 80.*

**Mixed Unit Addition and Subtraction:** A method for adding and subtracting in problems with different units of measurement.  
*For Example:* feet and inches, months and years, minutes and hours, pounds and ounces, etc…  
*See pages 26 – 27, 57 80.*

**Multiplication Facts:**

<table>
<thead>
<tr>
<th>Multiplication Facts</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anything times Zero is Zero and Zero times Anything is Zero.</td>
<td>N x 0 = 0 and 0 x N = 0</td>
</tr>
<tr>
<td>Any Number times One is that Number and One times Any Number is that Number</td>
<td>1 x N = N and N x 1 = N</td>
</tr>
</tbody>
</table>

*See pages 97, 185.*

**Notations for Division:** See Division Notations.

**Order of Operations aka P E (M/D) (A/S):** Multi-step problems with many different operations are to be done in a specific order.

**Step 1:** P stands for (Parentheses), [brackets], or {braces} - (grouping symbols) - Any mathematical operations inside grouping symbols are evaluated first.  
*Example:* $8 \times (3 + 0.1) = 8 \times 3.1 = 24.8$

**Step 2:** E stands for Exponents - Any numbers with exponents (including square root symbols) are done second.  
*Example:* $4 + 0.2^3 = 4 + 0.008 = 4.008$  
$5 + \sqrt{0.36} = 5 + 0.6 = 5.6$

**Step 3:** M/D stands for Multiplication and Division - Any multiplication and division problems are done third, working in order from left to right. If you see multiplication first (on the left) do that, but if division is first (on the left) do that.  
*Example:* $42 \div 7 \times 6 = 6 \times 6 = 36$  
$3 \times 4 \div 2 = 12 \div 2 = 6$

**Step 4:** A/S stands for Addition and Subtraction – Adding and Subtracting are done are done last, working in order from left to right. If you see addition first (on the left) do that, but if subtraction is first (on the left) do that.  
*Example:* $11 - 3 + 8 = 8 + 8 = 16$  
$5 + 6 - 11 = 11 - 11 = 0$

*See pages 175, 353.*

**PEMDAS:** See **Order of Operations** above.

**Percent Decrease/Increase:** Percent Decrease/Increase is the proportional amount by which an original number is decreased or increased.  
*See pages 421, 429.*
**Place Value:** Each non-zero digit has a value that depends on its place in the number.

*For Example:* The value of the digit 5 in 537 is 500 and its value in 375 is 5. (Think money!)

*See pages 4-5, 313.*

**Reciprocal/Finding a Reciprocal:** To find the reciprocal of any number (except 0) put the number in fraction form and then switch the numerator and denominator (or “flip it”).

*See pages 239, 297.*

**Reducing Fractions to Lowest Terms:** A fraction is in lowest terms when the numerator and the denominator have no prime factors in common. To reduce a fraction algebraically, remember to Factor, Cancel, Multiply.

*See pages 219 - 220.*

**Related Addition and Subtraction Sentences:** Using a given addition sentence there are two related subtraction sentences, both starting with the sum.

*For Example:* From the addition sentence $8 + 7 = 15$ we can write $15 - 8 = 7$ and $15 - 7 = 8$.

*See page 45.*

**Related Multiplication and Division Sentences:** Using a given multiplication sentence there are two related division sentences, both starting with the product.

*For Example:* From the multiplication sentence $8 \times 7 = 56$, where 56 is the product, we can write:

$$56 \div 8 = 7 \quad \text{and} \quad 56 \div 7 = 8$$

*See pages 124, 186.*

**Renaming Equivalent Fractions:** Equivalent fractions are two fractions that appear different but have the same numerical value. To create an equivalent fraction from an existing fraction multiply it by 1 written as \(\frac{2}{2}\), \(\frac{3}{3}\), \(\frac{4}{4}\), etc. When you multiply the existing fraction by 1 (in fractional form) you multiply the numerator and denominator by the same number and build a new equivalent fraction.

*See pages 215 - 216.*

**Reverse Average** is finding a specific test score required to achieve a desired average grade for a course.

*See pages 139, 186.*

**Rounding:** Determining an approximate value of a number based on its proximity to numbers above and below it in a specific place value.

*For Example:* 752 is closest to 750 when counting by tens.

*See pages 39 - 40, and 337.*

**Sharing and Cutting Problems:** See Cutting/Separating into Equal Size Pieces.

**Testing for Equal Fractions:** To see if two fractions are equal or if two ratios are proportional, cross multiply. If the cross products are equal then the fractions are equal or the two ratios are proportional OR check to see if the quotients are identical when doing top \(\div\) bottom.

*See pages 217, 297.*

**Total Value Problems:**

\[(\text{value for one item}) \times (\text{total number of items}) = (\text{total value of all the items together})\]

*See pages 145, 146.*

**Unit Cost/Unit Price:** Unit Cost is found by the formula:

$$\text{Total Cost} \div \text{Total Number of Units} = \text{Cost for One Unit}$$

*See page 149.*
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### Multiplication Tables

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